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A Note on Renewal Processes with ν-infinitely Divisible Interarrival Times

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ABSTRACT We derive an expression for the probability mass function of a renewal process whose interarrival times are positive independent random variables with a commom ν-infinitely divisible distribution. This generalizes (and corrects) a previous result of Pillai [17] concerning renewal processes with geometric exponential inter-arrival times.

Keywords Geometric exponential distribution; Geometric infinitely divisible distribution; Geometric stable distribution; Mittag-Leffler distribution; Poisson process; Scale mixture.

1. Introduction and Statement of the Main Result

Let X be a non-negative infinitely divisible (ID) random variable (RV) with the Laplace transform (LT) $\phi(s) = \exp(-\eta(s)), s \geq 0$. Then for each $t \geq 0$ the function

$$\phi_t(s) = \exp(-t\eta(s)), \quad s \geq 0$$

is a LT of an ID distribution. This is the distribution of the RV $X(t)$, where $\{X(t), t \geq 0\}$ is the Lévy process corresponding to X. Now, if ν is a non-negative RV with the cumulative distribution function (CDF) F and LT λ, independent of $\{X(t), t \geq 0\}$, then the RV $Y = X(\nu)$ is also non-negative and its LT and CDF are given by

$$\varphi(s) = \int_0^\infty \phi_t(s) dF(t) = \lambda(\eta(s)), \mathrm{s} \geq 0$$

and

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A Class of Distribution-free Tests for a Special Two-sample Location Problem Based on Ranked Set Sample Data

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ABSTRACT Shetty and Umarani [13] proposed a class of distribution-free tests for a special two-sample location problem. This class of tests is extended to data obtained by using ranked set sampling (RRS). The performance of this class is compared to tests based on simple random sample (SRS) data in terms of Pitman asymptotic relative efficiency.

Keywords Special two-sample location problem; Distribution-free; Ranked set sampling; Pitman asymptotic relative efficiency.

1. Introduction

When the measurement of experimental units is very expensive, time-consuming or destructive, there may be other informal measurements available that are far cheaper than formal, expensive full measurements. These informal measurements can be used to create a sampling design to produce a data set with smaller sampling variation than the simple random sampling. Ranked set sampling is a technique introduced by McIntyre [5]. He utilized large number of informal measurements to decide which expensive units should be fully measured. One can also refer to Kaur, Patil, Sinha and Taillie [3] for a review.

Testing the equality of location parameters in a two-sample problem has attracted the

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Laguerre Polynomial Approximations for Interval Estimation and Prediction for Exponential Distribution Based on Doubly Type-II Censored Data

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ABSTRACT In this paper, we make use of an orthogonal polynomial density approximation in order to obtain interval estimation for the scale parameter as well as prediction intervals for an exponential distribution based on doubly Type-II censored samples. We first obtain the exact moments of the pivotal quantity based on the Best Linear Unbiased Estimator (BLUE) of the scale parameter and then the density approximants expressed in terms of Laguerre polynomials are proposed, based on these exact moments. A comparative study of the interval estimation and the prediction intervals obtained by using the proposed approximation method to the corresponding exact numerical results of Lin and Balakrishnan ([15], [16]) is carried out. This reveals that the proposed moment-based approximation technique provides very accurate interval estimation as well as prediction intervals. Finally, we present some examples to illustrate all the methods of inference discussed here.

Keywords Exponential distribution; Moments; Laguerre polynomial approximants; Doubly Type-II censored data; Best linear unbiased estimator.

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Interval Estimation of $P(Y < X)$ in the Normal Case

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ABSTRACT We consider the problem of interval estimation of the stress-strength reliability $R = P(X < Y)$ where X and Y have independent normal distributions. We considered several types of approximate and bootstrap intervals. Their performances are investigated using simulation techniques and compared in terms of attainment of the nominal confidence level, symmetry of lower and upper error rates, and expected length. Recommendations concerning their use are given.

Keywords Bootstrap; Interval estimation; Normal distribution; Stress-strength model.

1. Introduction

Let X and Y be independently normally distributed random variables with means and variances are μ_x, σ_x^2 and μ_y, σ_y^2 respectively. Consider the quantity

$$R = P(Y < X) = \Phi\left(\frac{\mu_x - \mu_y}{\sqrt{\sigma_x^2 + \sigma_y^2}}\right) = \Phi(\delta)$$

(say), where Φ is the cumulative distribution function of the standard normal distribution. This quantity arises naturally in the context of mechanical reliability of a system with strength X and stress Y. The system fails any time if its strength is exceeded by the stress applied to it. Another interpretation of R is that it measures the effect of the treatment when X is the response for a control group and Y refers to the treatment group. The problem of developing confidence intervals for the stress-strength probability R in the normal case has received a wide attention in the literature. Owen *et al*. [10] considered the special case when $\sigma_x^2 = \sigma_y^2$ with equal sample

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Estimation of System Survival Function with Specified Number of Shocks

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ABSTRACT A two-component parallel system is subjected to a sequence of shocks from single source. Shocks are of two types, namely effective shocks and non-effective shocks. Damages due to effective shocks are exponential random variables. If the accumulated damage exceeds the threshold of the component, the component fails. The threshold of the component is also an exponential random variable. The survival probability of the system with k_0 (known) shocks is derived. The maximum likelihood estimator (MLE) and the uniformly minimum variance unbiased estimator (UMVUE) of the survival probability are obtained.

Keywords Two-component parallel system; Survival probability; Shock model; Life testing experiment; MLE; UMVUE.

1. Introduction

Problems in reliability theory are widely varied in nature. Consider the problem in which, the systems of various types functioning without failure is desired. But, at the same time, we cannot be sure of systems not being exposed to shocks or accidents during the period of operation. So, it is necessary to develop the mathematical methods to answer the questions like "with what probability can the system survive k_0 shocks", where k_0 is a pre-specified positive integer.

Shock models were developed for just this purpose. Esary *et al*. [6] studied some models of the life distribution of a component, subjected to random shocks occurring according

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Testing the Significance of Mean from Arbitrary Populations

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ABSTRACT In this article we consider a parametric method for testing the significance of mean by taking into consideration the effects of both skewness and kurtosis of the population. For a fixed level of significance, this method provides more powerful test compared to other existing ones, irrespective of the sample size. As the proposed method is free of any specific distributional assumptions, it can be used for testing the significance of mean from any population with a unimodal continuous distribution.

Keywords Cumulants; Edgeworth expansion; Generalized lambda distribution; Parametric tests; Transformation methods.

1. Introduction

Once sample data has been gathered through an observational study, statistical inference allows analysis to assess evidence in favor of some claims about the population from which sample has been drawn. The method of inference used to support or reject the claim based on the sampled data is known as test of significance. The special position, which the normal distribution held in conducting tests of significance of mean is mainly by virtue of central limit theorem. If m_1 is the mean of a sample of size n from a distribution having finite mean μ and variance σ^2 then the test of significance of mean is conducted based on the statistic $t = (m_1 - \mu)\sqrt{n}/\sigma$ as per Lindberg and Levy central limit theorem t is distributed as normal. When σ is unknown t is taken by replacing σ by its unbiased estimator, it then follows Student's t distribution. Hence the significance test of mean is generally conducted by

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Progressively Type-II Censored Sample from Geometric Life Time Model

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ABSTRACT Recently, a lot of interest is generated in the area of discrete lifetime models. When data in lifetimes are in grouped form, discrete models become very much handy in their analysis. In such context we consider the geometric distribution, a discrete analogue of the exponential distribution, as a basic discrete lifetime model. In the present paper, we study the maximum likelihood estimation of the parameters of geometric lifetime model and reliability function based on Type-II progressive censoring with changing parameters at each stage. We have derived also expected completion time of the i^{th} stage on the basis of complete observation as well as first n_s failures, $n_s < n_i$, $i = 1, 2, \ldots, K$, where K is the total number of stages for Type-II progressive censoring. Expected termination time of the test and expected total time of the test are also derived. A minimum sample size required for the test with given total cost of the experiment is obtained. A test for homogeneity of a selected set of parameters out of K parameters of the K-stage progressive censored lifetime model is also discussed. An illustrative example is cited.

Keywords Type-II progressive censoring, Maximum likelihood estimation, Geometric distribution, Expected termination time, Expected total test time, Sample size, Fixed total cost, Test for homogeneity.

1. Introduction

In life testing problems most of the work including Balakrishnan and Aggarwala [1] is based on continuous life time models only. A little work has been found in the literature based on discrete lifetime model. Recently, a lot of interest is generated in the area of discrete lifetime

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The Geometry of Translation Random Fields

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ABSTRACT In this paper we study the geometry of the translation random fields. We show that the excursion set of a translation random field above a high threshold has the same distributional properties as that of the Gaussian random field.

Keywords Translation random field; Excursion set; Convex component.

1. Introduction

Non-Gaussian random quantities are present in many fields in science and engineering such as wave elevation, wave forces, seismic ground acceleration, river flow, roughness of a road, geometric properties of a structure, etc. Such random responses have statistics that are inconsistent with the Gaussian distribution. If a Gaussian random field is considered to model such responses, then an unrealistic estimate of its reliability will be produced. So it is more suitable to find a non-Gaussian random field to fit such random responses. If $X(\mathbf{t}) \in S \subseteq R^D$ denotes a random field on S, then the simple way to find a non-Gaussian random field is to use the probability integral transform, i.e., to define a field $Y(\mathbf{t})$ by

$$Y(\mathbf{t}) = F^{-1} \circ \Phi(X(\mathbf{t})) = G(X(\mathbf{t})),\ \mathbf{t} \in S \tag{1}$$

Where F and Φ denote an arbitrary absolutely continuous distribution function and the distribution function of the standard Gaussian variable, respectively, and $\circ$ denotes the composition of functions. The field $Y(\mathbf{t})$ is called the *Translation Random Field*. We assume that F is twice differentiable.

The probability $P\{\sup_{\mathbf{t}\in S} X(\mathbf{t}) \geq x\}$ for large x is very important since it measures the reliability of a system in engineering. It is not possible to find its exact value in general.

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Intentionally Representative Sampling for Estimating a Population Mean

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ABSTRACT This paper introduces intentionally representative sampling (IRS), a novel sampling scheme that allows flexible use of prior and auxiliary information without introducing bias. IRS operates by allowing the user to create a universe of potential samples that excludes any samples considered unrepresentative. The good performance of IRS relative to simple random sampling and ranked-set sampling (RSS) is demonstrated via simulation studies using both simulated and actual data. One conclusion is that RSS is far from efficient in taking advantage of the auxiliary information that it assumes available.

Keywords Bias; Ranked-set sampling; Simple random sampling; Cluster sampling.

1. Introduction

Suppose that we wish to estimate the mean for some population. If this population consists of only a few units, each of which can be accessed and measured at negligible cost, then we might obtain a measurement for every unit in the population. This sort of census is typically not feasible, however. As a result, measurements can be obtained for only a limited number of units, and the quality of our estimate of the population mean depends on the particular sampling scheme chosen.

The basic sampling scheme that forms the basis for more complex schemes is simple random sampling (SRS), in which every possible sample is equally likely to be chosen. If prior information is available in the form of relevant covariates, then stratified random sampling tends to produce better estimates of the population mean than does SRS. If it is inexpensive to

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