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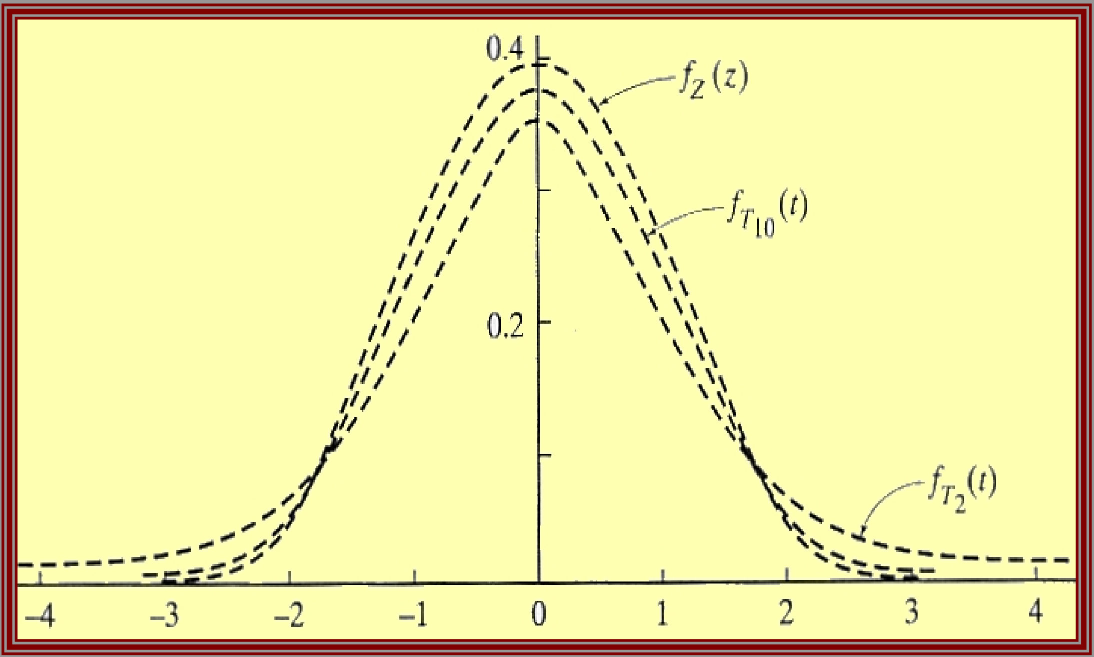
# JPSS

A comprehensive journal of probability and statistics
for theorists, methodologists, practitioners, teachers, and others

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A Comprehensive Journal of Probability and Statistics

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# *JPSS* Editorial Board

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**Aims and Scope** The ***Journal of Probability and Statistical Science*** (***JPSS***, ISSN 1726-3328) is a modified version of the ***Journal of Propagations in Probability and Statistics*** (***JPPS***, ISSN 1607-7083). ***JPSS***, like its predecessor ***JPPS***, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, and any other users of probability and/or statistics. The scope of ***JPSS*** is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of ***JPSS*** to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability and/or statistics.

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# Asymptotic Properties of a System Reliability Estimator under an Imperfect Debugging Model

Marcus A. Agustin and Ma. Zenia N. Agustin
*Southern Illinois University Edwardsville*

**ABSTRACT** This paper considers a series system of m softwares where the failure of each software follows a modified Jelinski-Moranda model. Upon system failure, the software that caused the failure is debugged. An imperfect debugging scheme is employed in the sense that the bug responsible for the failure is completely removed with a constant probability. At the conclusion of a fixed observation and debugging phase, tasks with random completion times are assigned to the system in order to assess system reliability. Estimators of relevant parameters associated with the failure process of each software as well as the system reliability are derived and asymptotic properties of the estimators are established.

***Keywords*** Competing risks; Maximum likelihood estimation; Software reliability.

## 1. Introduction

There have been a great deal of attention focused on the area of software reliability since the seminal paper by Jelinski and Moranda [11]. The Jelinski-Moranda model assumed that the failure rate of a software is proportional to the residual number of bugs, with each bug having a constant contribution to the failure rate. When the software fails, the bug that caused the failure is removed and no new bugs are introduced. Several models have been developed that either extended the assumptions of the Jelinski-Moranda model or considered various ways of modeling the failure rate of a software, as evidenced by the papers of Goel and Okumoto [8], Littlewood [15], van Pul [21, 22], Barendregt and van Pul [4], Agustin and Peña [2], Goseva-Popstojanova and Trivedi [9], Jeske and Pham [12], and Agustin [1], among others.

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# Estimation for Parameters of Feller-Pareto Distribution from Progressive Type-II Censoring and Some Characterizations

M. M. Mohie El-Din M. M. Amein H. E. El-Attar E. H. Hafez
Al-Azhar *University* *Helwan University*

**ABSTRACT** Feller-Pareto (FP) family is a very general unimodal distribution which includes a variety of distributions. In this paper, linear combination using approximate maximum likelihood and the best linear unbiased estimators from progressive Type-II censoring are performed to estimate the location and scale parameters of Feller-Pareto distribution. Moreover, relative efficiency technique is implemented to compare between the methods which were considered for estimation. Finally, some characterizations based on conditional distributions of order statistics for Feller-Pareto distribution are also obtained.

***Keywords*** Progressive Type-II censoring; Approximate best linear unbiased estimates; Approximate maximum likelihood estimates; Feller-Pareto distribution; Relative efficiency; Characterization.

## 1. Introduction

Experiments involving Type-II censoring are often used in life testing. Such tests are time and cost effective since it might take a very long time for all items to fail. A generalization of Type-II censoring is progressive Type-II censoring. In this case, the first failure in the sample is observed and a random sample of size $R_1$ is immediately drawn from the remaining $(n-1)$ unfailed items and removed from the test, leaving $(n-R_1-1)$ items in test. After the second item has failed, $R_2$ of the still unfailed items are removed, and so on. The

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# Asymptotic Properties of a Conditional Risk Function for Functional Data

Samir Benaissa Boubaker Mechab Abdelkader Bahram
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**ABSTRACT** In this paper, we investigate the asymptotic normality of the kernel density estimator (introduced by Ferraty and Vieu [4]) under dependent conditions of the conditional hazard function. The infill increasing setting is considered, that is when the covariates take values in some abstract function space. Our approch is based on the Doob's technique. It is shown that, under the concentration property on small balls of the probability measure of the functional estimator and some regularity conditions, the kernel estimate of the three parameters (conditional density, conditional distribution and conditional hazard) are asymptotically normally distributed.

***Keywords*** Asymptotic normality; Conditional hazard function; Functional Data; Non-parameter kernel estimation; Strong mixing.

## 1. Introduction

The conditional hazard rate arise in a variety of fields including econometrics, epidemiology, environmental science and many others. There is a large literature on the estimation of hazard function for independent as well as for dependent mixing data (see, for example, Watson and Leadbetter [12], Roussas [10], Estévez-Pérez *et al*. [1] and Quintela-del-Río [6]). However for the case of censored variables we can cite the work of Lecoutre and Ould-Said [8]. In recent years, there has been a considerable interest in functional data analysis. We refer to Ferraty and Vieu [4], Ferraty *et al*. [2], Ramsay and Silverman [9] and Laksaci *et al*. [5]. The work about the conditional hazard rate in infinite dimensional space for functional covariates are of Quintela-del-Río [7] and Ferraty *et al*. [3], where they introduce a kernel estimator and they prove some asymptotic properties

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# Katz Family of Distributions and Processes

K. K. Jose and K. D. Mariyamma
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**ABSTRACT** The Katz family of discrete distributions is revived and its statistical properties are explored. This family covers a wide spectrum including binomial, negative binomial, and Poisson distributions. A new stationary first order integer valued autoregressive process with Katz family of distributions is introduced based on the binomial thinning operator. Some properties of the process are established. The geometric Katz family of distributions is introduced and studied. Integer valued autoregressive processes corresponding to higher order generalizations are also discussed. A real data set is used to illustrate the application of the Katz family of distributions.

***Keywords*** Count data models; Discrete distributions; Discrete self-decomposability; Integer valued autoregressive processes; Statistical modelling.

## 1. Introduction

The Poisson distribution and the Negative binomial distribution are the most widely used discrete probability distributions for the analysis of count data. Bardwell and Crow [3] derived a two-parameter family of univariate hyper-Poisson distribution covering Poisson and the left truncated Poisson distributions as particular cases. The Katz families of distributions cover a wide spectrum including binomial, negative binomial, and Poisson distributions. A major motivation of Katz's [10] work was the problem of discriminating among binomial, negative binomial, and Poisson distributions when a given set of data is known to come from one or other of them. It has been used as a basis of developing more general families of distributions such as distributions defined by a discrete analogue to the Pearson system of continuous distributions studied in Ord [15, 16, 17]. Gurland and Tripathi [7] generalized the Katz's family to three and four parameter families of distributions, which covers the hyper-Poisson distribution as a special case (see Tripathi and Gurland [24, 25]). Fang [4, 5] developed Generalized method of moments (GMM) tests for the Katz family of distributions.

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# A Note on Uniform Strong Consistency of a Recursive Estimator of a Density Function

M. Samanta and S. Mandal
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**ABSTRACT** Davies [1] has proved the uniform consistency of a class of recursive estimators of a probability density function $f$ based on a random sample of size $n$ under the assumption that $f$ is uniformly continuous. In this paper, we show that the assumption of uniform continuity of $f$ is necessary for this type of convergence.

***Keywords*** Recursive kernel estimation; Uniform continuity; Uniform strong consistency.

## 1. Introduction

We consider a sequence $\{\mathbf{X}_n = (X_{1n}, X_{2n}, \ldots, X_{mn})\}$, $n$ = 1, 2, …, of independent and identically distributed $m$ dimensional vector random variables having a distribution function $F$. Let

$$g_n(\mathbf{x}) = g_n(x_1, x_2, \ldots, x_m) = n^{-1}\sum_{j=1}^{n} h_n^{-m} K(h_j^{-1}(\mathbf{x} - \mathbf{X}_j))$$

where $\{h_j\}$, $j$ = 1, 2, …, is a monotonically decreasing sequence of positive numbers such that

$$\lim_{n\to\infty} h_n = 0$$

and $K(\mathbf{u}) = K(u_1, u_2, \ldots, u_m)$ is a probability density function on $\mathcal{R}_m$, the $m$ dimensional convolution of the real line $\mathcal{R}_1$.

The properties of this estimate has been investigated by many researchers including Parzen [3], Nadaraya [2], Schuster [4, 5] and Van Ryzin [6] among others under the assumption that the distribution function $F$ has a density function $f$. When $n$ = 1, the following theorem regarding the convergence of $g_n(x)$ is well known.

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# Nonlinear Markov Chains and G-Brownian Motion

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**ABSTRACT** In financial mathematics, nonlinear models are ubiquitous. Recently, an ever increasing proportion of mathematical research is devoted to the analysis of nonlinear systems. However, little attention has been given to nonlinear Markov chains. In the paper, we develop theory of nonlinear Markov chains. We present general generators, diffuse generators and project generators for nonlinear Markov chains. Under such conditions, we study generation of G-Brownian motion on nonlinear Markov chains.

***Keywords*** Nonlinear Markov chains; Generators; G-Brownian motion.

## 1. Introduction

Nonlinear systems are widely used to model systems across science, management, and engineering, because most of real life problems involve nonlinear systems in nature. For example, nonlinear systems have been introduced to capture the salient features of underlying economic variables and to price derivatives of the securities in [2]. In recent years, people are interested in nonlinear Markov chains in nonlinear systems. We give a survey on the development of nonlinear Markov chains as follows.

Peng [5, 6] generated a kind of filtration-consistent nonlinear expectations through nonlinear Markov chains. Peng [7] first gave the definition of nonlinear Markov chains with pre-expectations in time homogeneous and use nonlinear Markov chains to generate nonlinear expectation. Nonlinear expectations as an alternative to mathematical expectations are being used extensively in robustness, finance and insurance literature of nonlinear systems. Xu and Zhang [10] constructed a G-Brownian motion by nonlinear Markov chain. Kolokoltsov [3] developed nonlinear Markov chains in discrete state and in continuous time and discrete space.

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# Bayesian Estimation of Rate Parameters of Queueing Models

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**ABSTRACT** In this paper we obtain the Bayes estimate of the arrival rate $\lambda$, service rate $\mu$ and traffic intensity $\rho$ of an M/M/1 queueing model using bivariate prior distribution of $(\lambda, \mu)$. The posterior distribution and credible sets for traffic intensity and predictive distribution of number in the system are also derived .

***Keywords*** Bayes estimate; Posterior distribution; Credible sets; Predictive distribution.

## 1. Introduction

Queues (or waiting lines) help facilities or businesses provide service in an orderly fashion. Forming a queue being a social phenomenon, it is beneficial to the society if it can be managed so that both the unit that waits and the one that serves get the most benefit. The ultimate objective of the analysis of queueing systems is to understand the behavior of their underlying processes so that informed and intelligent decisions can be made in their management. A major problem associated with the analysis of real life queueing data is the model selection. The great diversity of queueing problems gives rise to an enormous variety of models each with specific features. Incorporating more than one or two such features usually makes the model not only complicated but also analytically intractable. Therefore a substantial part of the literature deals with the models of a very simple structure. The Markovian assumption greatly simplify the modeling and solution.

After selecting an appropriate model for the queueing data, the next step is the estimation of performance measures. The common performance measures are traffic intensity ($\rho$), expected queue size ($L_q$), expected system size ($L$), expected waiting time ($W$) and expected waiting time in the queue ( $W_q$ ). All these quantities are functions of the queue parameters arrival rate ($\lambda$) and service rate ($\mu$).

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# On Predictive Estimation of Population Mean in Two-Stage Sampling

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A. K. Mangaraj — *Rajendra Junior College*
S. C. Senapati — *Ravenshaw College*

**ABSTRACT** Under the usual prediction approach advocated by Basu [2], we present a class of estimators for the finite population mean availing information on an auxiliary variable in a two-stage sampling procedure. The proposed class covers the regression-type estimator developed by Sahoo [5] as its minimum variance estimator.

***Keywords*** Asymptotic variance; Auxiliary variable; Prediction approach; Two-stage sampling.

## 1. Introduction and Prediction Criterion

Consider a finite population $U$, partitioned into $N$ first stage units (*fsu*) denoted by $U_1, U_2, \cdots, U_N$ such that the number of second stage units (*ssu*) in $U_i$ is $M_i$ and $M = M_1 + M_2 + \cdots + M_N$. Let $y_{ij}$ and $x_{ij}$ denote values of the study variable $y$ and an auxiliary variable $x$ respectively, for the $j^{\text{th}}$ ssu of $U_i (j = 1, 2, \cdots, M_i; i = 1, 2, \cdots, N)$. Define

$$\bar{Y}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} y_{ij} \quad \text{and} \quad \bar{X}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} x_{ij}$$

as the means of $U_i$, and

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} u_i \bar{Y}_i \quad \text{and} \quad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} u_i \bar{X}_i$$

as the overall population means, where $u_i = NM_i / M$. Assume that the variables $y$ and $x$ are

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# Asymptotic Properties of MLE's for Distributions Generated from a 2-Parameter Inverse Gaussian Distribution by a Generalized Log-Logistic Transformation

James U. Gleaton and M. Mahbubur Rahman
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**ABSTRACT** A generalized log-logistic (GLL) family of lifetime distributions is one in which any pair of distributions are related through a GLL transformation, for some (non-negative) value of the transformation parameter $\kappa$ (the odds function of the second distribution is the $\kappa$-th power of the odds function of the first distribution). We consider GLL families generated from a 2-parameter inverse Gaussian distribution (GLLIG families). It is shown that, for $\kappa$ > 3, the Maximum Likelihood Estimators (MLE's) for the parameters of the generated, or composite, distribution have the properties of strong consistency and asymptotic normality and efficiency.

***Keywords*** Generalized log-logistic families; Asymptotic properties; 2-parameter inverse Gaussian.

## 1. Introduction

Gleaton and Rahman [3] examined the asymptotic properties of the maximum likelihood estimators for parameters of a distribution generated from a 2-parameter Weibull distribution through a generalized log-logistic (GLL) transformation defined below. It was shown that, under certain restrictions on the parameter space, the distribution satisfied certain regularity conditions [11] that imply that the MLE's of the distribution parameters are asymptotically normal and efficient, and strongly consistent.

In an earlier paper, Gleaton and Lynch [2], discussed properties of lifetime distributions belonging to families generated by a GLL transformation:

$$G_\kappa(x) = l_\kappa \circ G(x) = [G(x)]^\kappa\{[G(x)]^\kappa + [\overline{G}(x)]^\kappa\}^{-1}, \quad \text{for} \quad x > 0,$$

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