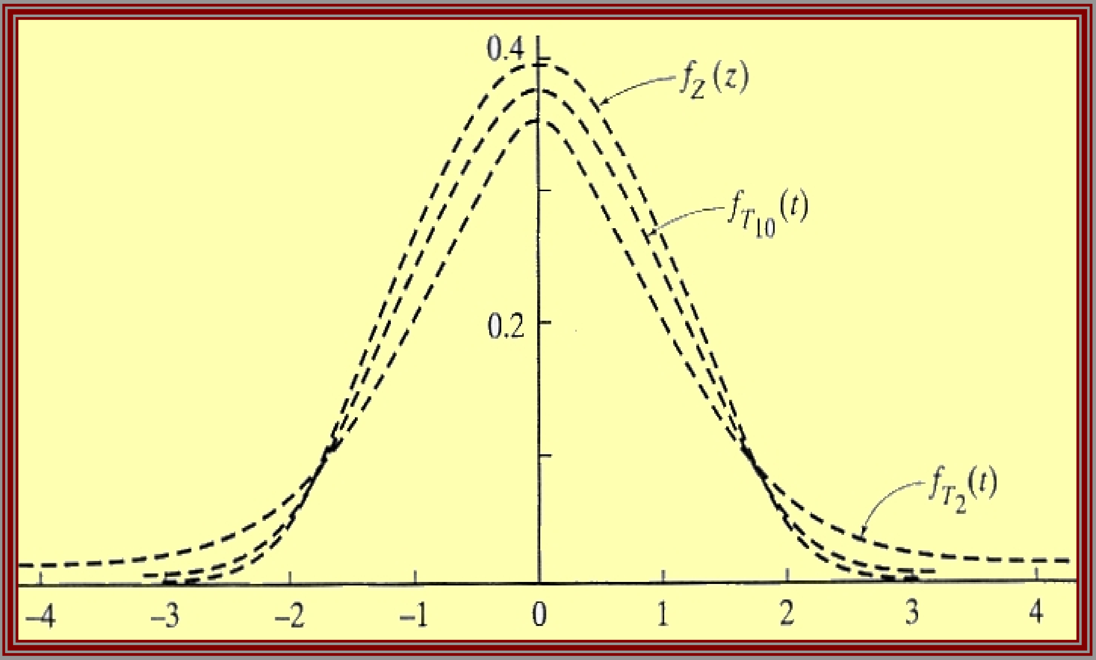
ISSN 1726-3328

JPSS

A comprehensive journal of probability and statistics
for theorists, methodologists, practitioners, teachers, and others



JOURNAL OF PROBABILITY AND STATISTICAL SCIENCE

Volume 11 Number 1 February 2013

ISSN 1726-3328

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Journal of Probability and Statistical Science

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Volume 11 Number 1

February 2013

JPSS Editorial Board

機統學誌 ***JPSS*** **ISSN 1726-3328**

Journal of Probability and Statistical Science

Published by: Susan Rivers' Cultural Institute, Hsinchu, Taiwan, ROC

(Cosponsored by: The International Chinese Association of Quantitative Management, Taiwan, ROC)

Aims and Scope The ***Journal of Probability and Statistical Science*** (***JPSS***, ISSN 1726-3328) is a modified version of the ***Journal of Propagations in Probability and Statistics*** (***JPPS***, ISSN 1607-7083). ***JPSS***, like its predecessor ***JPPS***, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, and any other users of probability and/or statistics. The scope of ***JPSS*** is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of ***JPSS*** to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability and/or statistics.

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魏蘇珊文教事業機構發行

總公司: 中華民國臺灣新竹市建美路 2 巷 26 號。

JPSS

Journal of Probability and Statistical Science

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Quadratic Forms in Unified Skew Normal Random Vectors

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ABSTRACT In this paper, we derive some moments of the quadratic forms and their functions under unified skew normal settings. First, we obtain the moment generating function of certain quadratic forms and discuss their distributions. Second, the joint moment generating functions of a linear compound and a quadratic form, and two quadratic forms have been obtained and conditions for their independence are given. Expected value of the ratios of quadratic forms in unified skew normal random vectors has also been studied. Finally, applications in context of time series and spatial statistics are discussed.

Keywords Unified skew normal distribution; Moment generating function; Linear compound; Independence; Ratio of quadratic forms; Time series.

1. Introduction

The multivariate unified skew normal (SUN) distribution has been introduced by Arellano-Valle and Azzalini [1]. In addition to being normal when the skewness parameter equal to zero, the family has properties similar to the normal distribution and yet is skew. We will study the quadratic forms and their applications in the context of SUN density. For other aspects of SUN density such as additive properties, inferential issues and applications, we refer to Gupta, Aziz and Ning [10], and Gupta and Aziz ([8], [9]).

There is a rich literature on the distribution of quadratic forms of the multivariate normal random vector. Earlier works were due to Cochran [5], Craig [6] etc. Recent references include

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□AMS 2000 Subject Classification: Primary 62 H10; Secondary 62P20.

 ISSN 1726-3328

Journal of Probability and Statistical Science
11(1), 17-30, Feb. 2013

Gumbel Distribution: Generalizations and Applications

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ABSTRACT As generalizations of Gumbel distribution, the Marshall-Olkin Gumbel maximum and minimum distribution as well as the Marshall-Olkin q-Gumbel distribution are introduced and their properties are studied. Minification processes with Marshall-Olkin q-Gumbel marginal distribution is also developed and studied.

Keywords Q-Gumbel distribution; Marshall-Olkin family; Extreme value distribution; Minification processes.

1. Introduction

Extreme Value Theory is widely used by many researchers in applied sciences when faced with modeling extreme values of certain phenomena, e.g., ocean wave modeling, wind engineering, thermodynamics of earthquakes, risk assessment on financial markets etc. The first results were developed considering independent observations but, more recently, models for extreme values have been constructed under the more realistic assumption of temporal dependence. The importance of the Gumbel distribution in practice is due to its extreme value behavior. It has been applied either as the parent distribution or as an asymptotic approximation, to describe extreme wind speeds, sea wave heights, floods, rainfall, age at death, minimum temperature, rainfall during droughts, electrical strength of materials, air pollution problems, geological problems, naval engineering etc.

Recently various authors have introduced several q-type distributions such as q-exponential, q-Weibull, q-logistic and various pathway models in the context of information theory, statistical mechanics, reliability modeling etc. The q-exponential distribution can be viewed as a stretched model (see Beck [2], Beck and Cohen [3]) for exponential distribution

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 ISSN 1726-3328

Journal of Probability and Statistical Science
11(1), 31-42, Feb. 2013

The Generalized Shannon-McMillan Theorems for Nonhomogeneous Markov Chains Indexed by a Homogeneous Tree

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ABSTRACT In this paper, a Shannon-McMillan theorem for the nonhomogeneous Markov chains field on a homogeneous tree is discussed by constructing a nonnegative martingale and analytical methods. As corollaries, some Shannon-Mcmillan theorems for the nonhomogeneous Markov chains field on a homogeneous tree and the nonhomogeneous Markov chain are obtained. Two results which have been obtained are extended.

Keywords Shannon-McMillan theorem; Homogeneous tree; Markov chain field; Relative entropy density.

1. Introduction

Let T be a homogeneous tree on which each vertex has $M+1$ neighboring vertices. We first fix any vertex as the "root" and label it by 0. Let σ, τ be vertices of a tree. Write $\tau \leq \sigma$ if τ is on the unique path connecting 0 to σ, $|\sigma|$ for the number of edges on this path. For any two vertices σ, τ, denote $\sigma \wedge \tau$ the vertex farthest from 0 satisfying

$$\sigma \wedge \tau \leq \sigma \quad \text{and} \quad \sigma \wedge \tau \leq \tau .$$

If $\sigma \neq 0$, then we let $\bar{\sigma}$ stand for the vertex satisfying $\bar{\sigma} \leq \sigma$ and $|\bar{\sigma}| = |\sigma| - 1$ (we refer

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□Received August 2010, revised June 2012, in final form August 2012.
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□Classification (MSC 2000): 60F15.
□The work is supported by Foundation of Anhui Educational Committee (NO. KJ2012B117).

 ISSN 1726-3328

Journal of Probability and Statistical Science
11(1), 43-58, Feb. 2013

On Correlated Random Graphs

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ABSTRACT Given $0 \le \rho \le 1$ and an integer *n* we construct a sequence of dependent *Bernoulli* random variables $\{\varepsilon_1, \ldots, \varepsilon_n\}$ on the probability space ([0, 1], *dx*) with $P(\varepsilon_i = 1) = p = 1 - P(\varepsilon_i = 0)$ and pairwise correlation ρ between ε_i and ε_j. As an application we introduce a class of random graphs with *n* nodes for which connectivity by an edge between any two nodes has probability *p* while interdependency between any two edges is characterized by correlation ρ. A construction of graphs with such properties along with a *Monte Carlo* algorithm for their simulation are given.

Keywords Dependent Bernoulli variables; Generalized binomial distribution, ρ-correlated binomial random graphs, Monte Carlo algorithm.

1. Introduction

Bernoulli trials play a central role in statistical analysis of modeling phenomena arising in science, engineering and graph-based random networks [7]. In the monographs [1], [2], [3] among others, {0, 1} valued independent random variables serve as building blocks in *binomial Random Graph* models, also studied in [4], [5], [6].

Our main objective is to relax independence by introducing a class of *dependent Bernoulli variables* leading to *Generalized Binomial Distribution*. The second objective is concerned with construction of ρ*- correlated binomial Random Graphs*. Finally, we devise an algorithm for generating graphs with *n* nodes and randomly placed edges. Simulations are carried for several illustrating examples. It is worth noting that correlation ρ in our model increases connectivity (i.e., higher probability for a larger total # of edges for $\rho > 0$). Also, $\rho = 0$ reverts back to the independent *Bernoulli* model, whereas the fully correlated case $\rho = 1$ gives *complete graph* with *C*(*n*, 2) edges with probability *p*, and an *empty graph* with no edges with probability $1 - p$ respectively.

□Received February 2012, revised October 2012, in final form January 2013.
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 ISSN 1726-3328

Journal of Probability and Statistical Science
11(1), 59-78, Feb. 2013

Estimating the Reliability Function for a Family of Inverse Distributions

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ABSTRACT A family of inverse distributions is proposed. The problems of estimating the reliability functions are considered. Uniformly minimum variance unbiased estimators and maximum likelihood estimators are derived. A comparative study of the two methods of estimation is done. Simulation study is performed.

Keywords Inverse distributions; Reliability function; Uniformly minimum variance unbiased estimators; Maximum likelihood estimators; Simulation study.

1. Introduction

A lot of work has been done in the literature to deal with inferential problems related to various inverse distributions. Keller and Kamath [20] derived inverse Weibull distribution on the basis of physical considerations on some failures of mechanical components subject to degradation phenomena, such as wear, fatigue and corrosion. Keller, Giblin and Farnworth [21] exhibited that this distribution gave best fit to the failure data of dynamic engine components (pistons, crankshaft, main bearings, etc.). Mudholkar and Kollia [28] referred to it as the reciprocal Weibull distribution. Erto [16] showed that the inverse Weibull distribution gave a good fit to the life testing data sets such as the times to breakdown of an insulating fluid subject to the action of a constant tension, reported in Nelson [29] and to the survival data in cancer chemotherapy, reported in Carter, Wampler and Stablein [8]. Such analysis led to further study of some estimation procedures and statistical properties of estimators. Erto [16] considered plotting positions technique and least-squares method to obtain the estimators for parameters and reliability function of the inverse Weibull distribution. Calabria and Pulcini [4] investigated the statistical properties of the maximum likelihood

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 ISSN 1726-3328

Journal of Probability and Statistical Science
11(1), 79-95, Feb. 2013

Generalized Inference for the Common Location Parameter of Several Location-Scale Families

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ABSTRACT In this paper, we are interested in inference problem concerning the common location parameter of k location-scale families with $k \geq 2$. More specifically, we study the case where the scale parameters of the families are unknown and possibly heterogeneous. The proposed solution is derived by using generalized inference method. To this end, we present a method of constructing the required generalized pivotal quantity (GPQ) and generalized p-value (GPV) for the common location parameter. The proposed approach is based on the minimum risk equivariant estimators (MRE) which is more general and more efficient than the maximum likelihood estimators (MLEs). Thus, we extend the approaches based on MLEs and conditional inference which have been so far applied to some specific distributions. Also, with intensive simulation studies, we illustrate the performance of the proposed approach in small and moderate sample sizes. Finally, the approach is applied to analyse the normal body temperature.

Keywords GCIs; Generalized p-value; Location-scale family; MRE; Pitman estimator.

1. Introduction

Testing the common location parameter of several location-scale families with unknown scale parameters is one of the most interesting statistical inference problems. There are many applications in which this problem is involved. For instance, this situation arises in statistical analysis that combines the information from several independent studies or meta-analysis. Indeed, the meta-analysis is more frequent in clinical trials as well as in social and behavioral sciences. Also, this is commonly seen in many statistical areas or designs such as balanced incomplete block designs, panel models, and some regression models, and in each of these scenarios, practitioners are often interested in inference concerning the common location parameter of several distributions with unknown scale parameters. For other applications and scenarios, we refer to Krishnamoorthy and Lu [6].

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 ISSN 1726-3328

Journal of Probability and Statistical Science
11(1), 97-110, Feb. 2013

Bayes Prediction Intervals for the Pareto Model

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ABSTRACT The Pareto model is proposed to be underlying model from which observables are to be predicted under the Bayesian approach. The prediction has been done under one-Sample and two-Sample Bayes prediction techniques. Both one-sided interval and the interval of central coverage prediction limits have been obtained. The item failure censored data from the proposed model is considered and the performances of the procedures are illustrated by a simulation technique.

Keywords Bayesian prediction technique; Item failure censoring; One and two samples prediction techniques.

1. Introduction

The Pareto distribution and their close relatives provide a very flexible family of fat-tailed distributions, which may be used as a model for the income distribution of higher income group. The Pareto model plays an important role in socio-economic studies. It is often used as a model for analyzing areas including city population distribution, stock price fluctuation, oil field locations and military areas. It has been found to be suitable for approximating the right tails of distribution with positive skewness. The Pareto distribution has a decreasing failure rate, so it has often been used for model survival after some medical procedures (the ability to survive for a longer time appears to increase, the longer one survives after certain medical procedures).

Davice & Feldstien [6] have viewed the Pareto distribution as a potential model for the life testing problems. This distribution has established its important role in variety of other problems such as size of cities and firms (Steindle [19]), business mortality (Lomax [11]), service time in queuing system (Harries [9]). Freiling [8] applied the Pareto law to study the distributions of nuclear particles. Harries [10] used this distribution in determining times of

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Received September 2011, revised July 2012, in final form October 2012.

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 ISSN 1726-3328