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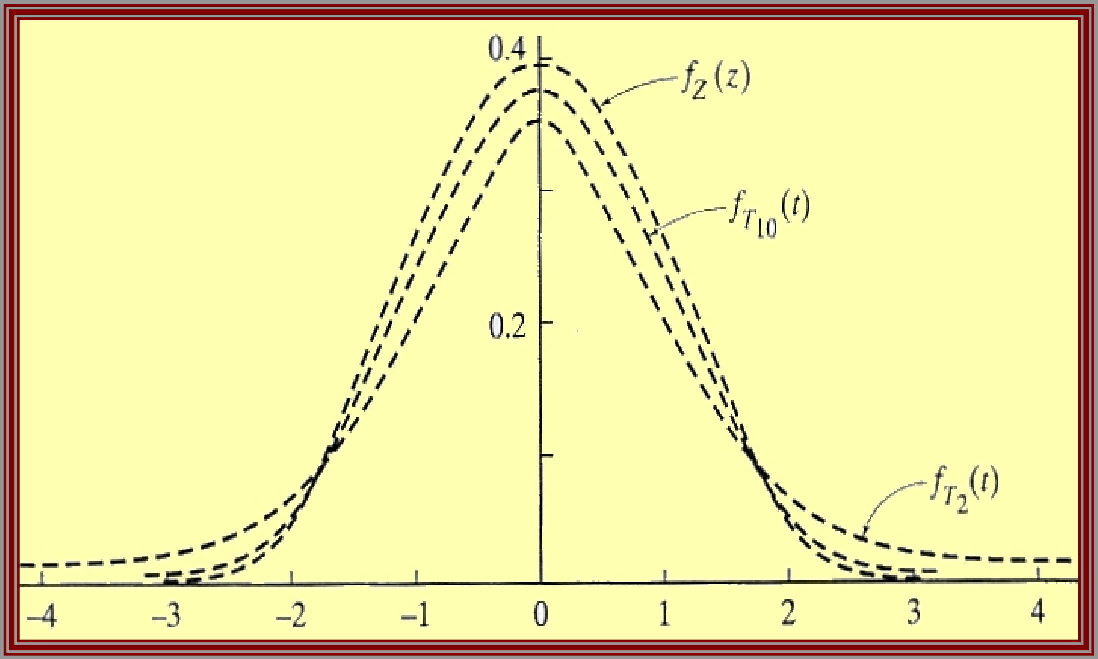
A comprehensive journal of probability and statistics
for theorists, methodologists, practitioners, teachers, and others



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10(2), 111-124, Aug. 2012

Mean Maximum Difference between Simple Random Walks

David K. Neal
Western Kentucky University

ABSTRACT Let X and Y be simple random walks with initial heights j_X and j_Y where $j_X < j_Y$. The processes independently move upward or downward 1 unit at a time on each simultaneous step with the probability of X moving upward being p_X and the probability of Y moving upward being p_Y . For $p_Y \le p_X$, with $p_Y \neq 1$ and $p_X \neq 0$, we derive the average value of the maximum distance between the processes through the time of paths colliding. Then for arbitrary p_X and p_Y , we derive the expected values of the maximum and minimum of the difference in heights through n steps, and show the relationship between these average extrema and the average heights of X and Y after n steps.

Keywords Simple random walk; Average extrema; Average maximum difference.

1. Introduction

Two random walks X and Y begin with different initial heights and independently move at random either up or down one unit at a time on each simultaneous step. Individual pairs of paths of X and Y may drift apart and thus never attain the same height. But under certain conditions, pairs of paths of X and Y will collide almost surely, and the average time of collision will be finite. In this case, the maximum distance between paths will have finite expectation. In this article we shall derive the expected value of this maximum distance. Then under general conditions, we derive the expected values of the maximum and minimum of the difference in heights through n steps. Finally we show the relationship between these average maximum and minimum values and the average heights of the paths after n steps.

2. Initial Conditions

We assume that X and Y begin at integer heights j_X and j_Y , respectively, where $j_X < j_Y$. We first assume that $j_Y - j_X$ is even so that it is possible for X and Y to attain the same height at the

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Perturbation Functions for Exponential-Order M-indeterminate Distributions

Jiantian Wang
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ABSTRACT In this paper, we introduce a family of distributions, refereed to as exponential-order distributions, which includes some classical M-indeterminate distributions, such as log-normal distributions and powers of inverse Gaussian distributions. For this kind of M-indeterminate distributions, we systematically present some methods for finding the associated perturbation functions.

Keywords M-determinacy, Perturbation function, Problem of moments, Stieltjes class.

1. Introduction

For a positive continuous random variable X, let $F(x) = P(X < x)$ be its cumulative distribution function. We say F is M-finite if all its moments

$$m_k = E(X^k) = \int_0^\infty x^k dF(x),\ k = 1, 2, \cdots$$

are finite. The classical problem of moments is: Is F the only distribution corresponding to the moment sequence $M = \{m_k\}$, $k = 1, 2, \cdots$? If it is, we say the problem of moments has a unique solution, or F is M-determinate. Otherwise, we say F is M-indeterminate. When F is M-indeterminate, then there is at least one distribution, say G, such that $G \neq F$ and G has the same moment sequence as that of F. In this case we say F and G are moment-equivalent.

Berg [3] shows that: If F is M-indeterminate, then there are infinitely many distributions all having the same moments as those of F. However, this result does not tell us what these distributions are. Hence, it is desirable to illustrate the M-indeterminacy of F by showing explicitly those other distributions which are moment equivalent to F.

For distribution F, denote $f(x) = F'(x)$ as its density function. For density function $f(x)$,

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Esscher Transformed Laplace Distributions and Its Applications

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ABSTRACT In this article, we introduce a new class of asymmetric Laplace distributions through Esscher transformation introduced in Esscher [8], namely Esscher transformed Laplace distribution. It is a sub-class of one parameter exponential family and an alternative to various types of asymmetric Laplace distributions given in Kozubowski and Podugorski [14]. We derive their representations and obtain explicit forms for their densities, distribution functions and quantile functions. Properties of the distribution are derived and estimation methods are discussed. The distribution exhibits maximum entropy property. The probability density functions in this class satisfy several properties compared to the general class of asymmetric Laplace distributions. We consider the problem of testing whether the population belonging to the classical Laplace distributions against the alternative that it belongs to the Esscher transformed Laplace distributions or asymmetric Laplace distributions. Finally an application of Esscher transformed Laplace distribution in financial modeling is also considered and its goodness of fit compared to the asymmetric Laplace distribution is measured.

Keywords Esscher transformation; Entropy, Financial modeling; Maximum likelihood estimator; Moment estimator; One parameter exponential family.

1. Introduction

For many years the Laplace distribution was a popular topic in probability theory due to its application in modeling phenomena with “heavier than normal tails”, see, [6] and [12]. Asymmetric Laplace distributions, a skewed generalizations of the classical Laplace law has got considerable attention among the researchers working in financial modeling, engineering, science, share market return models, stochastic variance models and time series modeling. The

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Note on the Functional Linear Estimate of Conditional Cumulative Distribution Function

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ABSTRACT In this paper we introduce a new nonparametric estimation of the conditional cumulative distribution of a scalar response variable Y given a functional random variable X. Our estimate is based in local linear approach. We prove under general conditions, the almost complete convergence (with rate) of the construct estimate.

Keywords Local linear estimation; Conditional cumulative distribution; Functional random variables; Semi-metric space; Small balls probability.

1. Introduction

Recently, the statistical modelization of functional data have received a growing attention. This great consideration is motivated by the progress of the informatics tools permits the recuperation of large data sets, available essentially by real time monitoring, and computers can manage such databases. Functional data statistic (see Bosq [4], Ramsay and Silverman [12], for the parametric model, Ferraty and Vieu [10], for the nonparametric case) can help to analyze such data sets. In this work, we consider the problem of the estimation of the cumulative distribution function of real random variable Y conditioned by functional random variable X by using a local linear approach.

Noting that, the estimation of the conditional cumulative distribution function has great importance. It is involved in many applications, such as reliability or in survival analysis. Moreover, in prediction problem's, there are several prediction tools in nonparametric statistic, such the conditional mode, the conditional median or the conditional quantiles, are based on the preliminary estimate of this nonparametric model. The literature on this topic is quite important, (see, for instance, Roussas [13] and Stute [14], Deheuvels and Mason [5] for non functional case

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Sequential Estimation of Two-dimensional Sinusoidal Models

Anurag Prasad Debasis Kundu Amit Mitra
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ABSTRACT Estimating the unknown parameters of a two-dimensional (2-D) sinusoidal signal is an important and a difficult problem. In this paper, we propose a simple sequential procedure for estimating the unknown frequencies and amplitudes of the 2-D sinusoidal components when the signal is affected by noise. When there are p components in the signal, the k-th step of the procedure provides strongly consistent estimators of the k dominant sinusoids when $k \le p$. When $k > p$, the supernumerary amplitude estimators converge to zero almost surely. The asymptotic distribution of the proposed estimators coincides with the asymptotic distribution of the least-squares estimators. Numerical simulations are performed for various sample sizes and for various model parameters. Some real texture data and some synthesized texture data are analyzed.

Keywords Frequencies; Amplitudes; Least-squares estimators; Strongly consistent estimators; Asymptotic distributions; Bayesian Information Criterion.

1. Introduction

In this paper we consider the problem of estimating the parameters of the following 2-D sinusoidal signal;

$$y(m,n) = \sum_{k=1}^{p} \{A_k^0 \cos(m\lambda_k^0 + n\mu_k^0) + B_k^0 \sin(m\lambda_k^0 + n\mu_k^0)\} + X(m,n). \qquad (1)$$

Here A_k^0 and B_k^0 are unknown real numbers, usually known as amplitudes, λ_k^0, $\mu_k^0 \in (0,\pi)$ are unknown frequencies. The additive error $\{X(m, n)\}$ is from a stationary random field and satisfies Assumption 1, which will be described later. The number of components 'p' may be known or unknown. The problem is to estimate the unknown parameters given a sample $\{y(m, n);\ m = 1, \ldots, M,\ n = 1, \ldots, N\}$.

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Inference in Generalized Linear Model with Cluster Structure in Covariates

Pengfei Guo, Xiaogang Wang, Yuehua Wu, and Hanfeng Chen
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ABSTRACT The relevant weighted likelihood method is a very useful tool to modify the classical maximum likelihood when the observations are not identically distributed. The authors propose to extend this method to generalized linear models when the covariates are generated from different populations. Furthermore, a general and effective method is proposed to choose the weights using the estimated probability of membership through the expectation-maximization algorithm. The proposed weights are applicable to both discrete and continuous covariate variables and work well in dealing with the cluster structure in generalized linear models. The asymptotic properties of the estimator are derived. The simulations studies suggested that the weighted likelihood equations with proposed weights are more powerful in testing than the classical methods. Applications to real data sets have also implied that the method could detect real relationships that the classical methods failed to discover.

Keywords EM algorithm; Generalized linear model; Model-based clustering; Weighted likelihood estimation.

1. Introduction

Generalized linear models have been widely used in practice. McCullagh and Nelder [10] provided an excellent comprehensive introduction with detailed discussions. Although the classical generalized linear models are quite powerful and enjoy many desirable asymptotic properties, they rely explicitly or implicitly on the model assumptions. One underlying assumption is that the data generating mechanism is homogeneous for all data points. To be more specific, the parameters remain constant for different domains of the covariates. This assumption, however, can be violated in practice.

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Inference on $P(X_2 < X_1)$ in Skewed Log-Laplace Distribution

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University of Mumbai

ABSTRACT In this paper, we have assumed that X_1 and X_2 are continuous and independent random variables following skewed log Laplace distribution. The probability that a random variable X_2 is less than X_1 is calculated for various forms of skewed log Laplace distribution. The maximum likelihood estimator of $R = \Pr(X_2 < X_1)$ is obtained. The Bias and MSE of R based on simulated samples are provided. The confidence interval for R is derived by obtaining the distribution of R using invariance property of maximum likelihood estimators.

Keywords Skewed Laplace distribution; Skewed log-Laplace distribution; MLE.

1. Introduction

Over the past many years, log-Laplace models appeared sporadically in science literature. Originally its symmetric form and later the asymmetric model which is known as skewed log-Laplace distribution was used for modeling various phenomena by a number of researchers. Inoue [1] derived the symmetric log-Laplace distribution from stochastic model for income distribution. Log-Laplace models have been recently proposed for growth rates of diverse processes such as annual gross domestic product, stock prices, interest or foreign currency exchange rates, company sizes and other processes. Skewed log-Laplace distribution has been used extensively as a model for particle size data, which plays an important role in disciplines as diverse as archaeology (e.g., quartz inclusions in the fabric of pottery), fuel technology (e.g., droplets of rocket propellant), medicine (e.g., blood cells), and geology (e.g., grains of sand) (see, Kozubowski and Podgorski [2]). In context of fuel technology, if X_2 represents the maximum chamber pressure generated by ignition of a solid propellant and X_1 represents the strength of the rocket chamber, then $R = \Pr(X_2 < X_1)$ is the probability of successful firing of the engine. Because of such applications, the calculation and the estimation of $R = \Pr(X_2 < X_1)$ are important.

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Addendum to “Asymptotic Expansion of the Coverage Probability of James-Stein Estimators”, *Theory of Probability and Its Applications*, 51(4) (2007), 683-695.

Kamon Budsaba and Sujitta Suraphee
Thammasat University

ABSTRACT In this note we provide complete proof of one important statement that has been presented without proof in the article by S. E. Ahmed, A. K. MD. E. Saleh, A. I. Volodin, and I. N. Volodin entitled “Asymptotic expansion of the coverage probability of James-Stein estimators” published in *Theory of Probability and Its Applications*, **51**(4) (2007), 683-695.

Keywords Multivariate normal distribution; Chi-square distribution; Joint distribution; Characteristic function.

1. Introduction

In paper [1] the authors presented Proposition 1 without proof. We consider this proposition as very important because nearly all statements in paper [1] are based on it. Moreover, we found that it's proof is not so easy as the authors of [1] may think. Hence in this note we present the detailed proof of Proposition 1 formulated in paper [1]. Before formulating Proposition 1 we introduce some notation.

Let $\boldsymbol{Z} = (Z_1, Z_2, \cdots, Z_p)'$ be a p-dimensional normally distributed random vector such that each Z_j, $1 \le j \le p$ has standard normal distribution and they are independent of each other. That is, $\boldsymbol{Z}$ is p-dimensional normal random vector with zero mean and the covariance matrix equals to $p \times p$ identity matrix. Let $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_p)'$ be a vector of constants.

Now we introduce two random variables X and Y as

$$\boldsymbol{X} = \|\boldsymbol{Z}\|^2 = \boldsymbol{Z}'\boldsymbol{Z} = \sum_{j=1}^{p} Z_j^2,$$

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Some Randomized Response Models

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ABSTRACT Two randomized response models are presented in this article. Motivated by Franklin's [4] RRM, an extension of Warner's [10] randomized response model (RRM) is presented. An unrelated trait RRM whose impetus is acquired from Mangat and Singh [6] is also presented. In the unrelated trait RRM, both the cases of known and unknown proportion of unrelated trait are studied. It has been observed that when the probability of selection of sensitive question, say P, in Warner's [10] model is greater than 0.5, the proposed models work more efficiently than Warner's [10] RRM. In case of unknown proportion of unrelated trait the second proposed model performs well compared to Warner's [10] RRM when $|0.5 - P|$ is smaller. The proposed RRMs are likely to provide more privacy protection and anonymity. Therefore, these RRMs are more likely to result in greater cooperation form survey respondents.

Keywords Evasive answer bias; Estimation of proportion; Randomized response model.

1. Introduction

It is an easy consequence of direct survey that while asking about sensitive items, the respondent, in general, misrepresent his/her true responses or repudiate to respond in any way. Its domino effect is biased estimation of the parameters. Warner [10] introduced an ingenious method to lessen the biasedness in the estimators and to boost the response rate. Warner's [10] model consists of two complimentary questions A and A^c to be answered on probability basis, where A is "do you possess the sensitive trait", and A^c is "do you not possess the sensitive trait". The two questions A and A^c are offered to respondents with preset probabilities P and $(1 - P)$, respectively. Using simple random sampling with replacement (SRSWR) sampling, the i^{th} selected respondent is asked to select a question (A or A^c) and report *yes* if his/her actual status matches with selected question and *no* otherwise.

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On the Choice of a Beta Prior Distribution for Binomial Sampling

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ABSTRACT The incorporation of prior information about a parameter into a statistical procedure is an essential feature of Bayesian statistics. However, the manner in which this is done is often arbitrary. In order to reduce such arbitrariness, methodology based on information theoretic concepts is introduced which (a) quantifies the amount of information provided by the sample data relative to that provided by the prior distribution and (b) allows for a ranking of prior distributions with respect to conservativeness, where conservatism refers to restraint of extraneous information which is embedded in any prior distribution of the parameter. To illustrate the implementation of the methodology, the most conservative beta prior distribution under a binomial sampling model based on the Rényi information is determined for three situations: (1) no prior estimate of θ where θ is the success probability, is available, (2) a prior point estimate of θ is available, and (3) a prior interval estimate of θ is available.

Keywords Conservative prior distribution; Information theory; Ranking of prior distributions; Rényi information measure.

1. Introduction

Suppose that sample data $\tilde{x}$ are obtained by observing the random vector $\tilde{X}$ whose sampling distribution is assumed to belong to some specified family of probability density functions (pdfs)

$$F = \{f_{\tilde{\theta}}(\tilde{x});\ \tilde{x} \in \chi,\ \tilde{\theta} \in \Omega\},$$

where χ is the sample space and Ω is the parameter space. The parameter $\tilde{\theta}$, which indexes the members of F is unknown. Under the Bayesian approach, $\tilde{\theta}$ is assumed to be a value taken on by the random vector $\tilde{\Theta}$ whose prior pdf is $g(\tilde{\theta})$. The essence of $g(\tilde{\theta})$ is to incorporate into

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