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Aims and Scope The ***Journal of Probability and Statistical Science*** (***JPSS***, ISSN 1726-3328) is a modified version of the ***Journal of Propagations in Probability and Statistics*** (***JPPS***, ISSN 1607-7083). ***JPSS***, like its predecessor ***JPPS***, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, and any other users of probability and/or statistics. The scope of ***JPSS*** is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of ***JPSS*** to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability and/or statistics.

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Volume 9 Number 2 August 2011

Table of Contents

Theory and Methods

Teaching and Applications

Appendix

Journal of Probability and Statistical Science
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Conditional and Unconditional Weak Laws of Large Numbers for Bootstrap Sample Means with Random Bootstrap Sample Sizes

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ABSTRACT For a sequence of random variables $\{X_i, i \geq 1\}$, both conditional and unconditional weak laws of large numbers are established for bootstrap samples where the bootstrap sample means have random bootstrap sample sizes. The results are very general in that the random variables $\{X_i, i \geq 1\}$ do not need to be independent or identically distributed or be of any particular dependence structure. Moreover, in general, no moment conditions are imposed on the $\{X_i, i \geq 1\}$ which are constrained solely by the random bootstrap sample sizes $\{M_n, n \geq 1\}$. The sharpness of the results is discussed, three illustrative examples are presented, and two open problems are posed. This paper extends work of Einmahl and Rosalsky [12].

Keywords Boostrap sample mean; Conditional and unconditional weak laws of large numbers; Random bootstrap sample size; Convergence in probability; Almost sure convergence.

1. Introduction

Bootstrap samples were introduced by Efron [11] for a sequence of independent and identically distributed (i.i.d.) random variables and for the case where the bootstrap sample size M_n coincides with the sample size *n*. An extensive and more general literature of investigation has emerged wherein the the bootstrap sample size can be a deterministic function of the sample size; see the article by Csörgő and Rosalsky [9] for a comprehensive survey of first-order limit theorems available for bootstrap sample sums.

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The Hájek-Rényi Inequality for $\mathcal{M}$-dependent Arrays and a General Strong Law of Large Numbers

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ABSTRACT We extend the Hájek-Rényi inequality to $\mathcal{M}$-dependent arrays and establish a general strong law of large numbers for double arrays.

Keywords $\mathcal{M}$-dependent array; Nondecreasing array; Hájek-Rényi inequality; Strong law of large numbers.

1. Introduction

Hájek and Rényi [4] proved the following important inequality: If $(X_j, j \geq 1)$ is a sequence of independent random variables with zero means and $(b_j, j \geq 1)$ is a nondecreasing sequence of positive real numbers, then for any $\varepsilon > 0$ and for any positive integers n; n_0 $(n_0 < n)$,

$$P\left(\max_{n_0 \leq k \leq n} \frac{1}{b_k}\left|\sum_{j=1}^{k} X_j\right| \geq \varepsilon\right) \leq \frac{1}{\varepsilon^2}\left(\sum_{j=1}^{n_0} \frac{E(X_j^2)}{b_{n_0}^2} + \sum_{j=n_0+1}^{n} \frac{E(X_j^2)}{b_j^2}\right).$$

This inequality is a generalization of the Kolmogorov inequality and is a useful tool to prove the strong law of large numbers. Fazekas and Klesov [1] gave a general method for obtaining the strong law of large numbers for sequences of random variables by using a Hájek-Rényi type maximal inequality. Afterwards, under the same conditions as those in Fazekas and Klesov, Hu and Hu [5] obtained the strong growth rate for sums of random variables. Prakasa Rao [10] extended the Hájek-Rényi inequality to associated random variables. Recently, Sung [12] improved the inequality of Prakasa Rao, and he used this result to obtain the integrability of supremum and strong law of large numbers for associated random variables. For some other results in this topic, see Klesov *et al.* [7], Liu *et al.* [8], Gan and Qiu [3], Hu *et al.* [6]. The aim of this paper is to extend the Hájek-Rényi inequality to $\mathcal{M}$-dependent arrays and establish a general strong law of large numbers for double arrays.

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On the Distribution of Extrema for a Class of Lévy Processes

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ABSTRACT Suppose X_t is either a regular exponential type Lévy process or a Lévy process with a bounded variation jumps measure. The distribution of the extrema of X_t play a crucial role in many financial and actuarial problems. This article employs the well known and powerful Riemann-Hilbert technique to derive the characteristic functions of the extrema for such Lévy processes. An approximation technique along with several examples is given.

Keywords Principal value integral; Hölder condition; Padé approximant; Continued fraction; Fourier transform; Hilbert transform.

1. Introduction

Suppose X_t be a one-dimensional, real-valued, right continuous with left limits (càdlàg), and adapted Lévy process, starting at zero. Suppose also that the corresponding jumps measure, ν, is defined on $\mathbb{R}\setminus\{0\}$ and satisfies

$$\int_{\mathbb{R}} \min\{1, x^2\}\nu(dx) < \infty .$$

Moreover, suppose the stopping time $\tau(q)$ is either a geometric or an exponential distribution with parameter q that is independent of the Lévy process X_t, and that $\tau(0) = \infty$. The extrema of the Lévy process X_t are defined to be

$$M_q = \sup\{X_s : s \le \tau(q)\}; \quad I_q = \inf\{X_s : s \le \tau(q)\}. \tag{1}$$

The Wiener-Hopf factorization method is a technique that can be used to study the characteristic function of M_q and I_q. The Wiener-Hopf method has been used to show that:

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On the Meaning of Parameters in Approximation Models

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ABSTRACT Models provide the foundation to statistics and are generally viewed as approximations to the underlying true state of nature. Data are used to estimate parameters in these approximating models. Assuming there exist a true underlying model, the parameters in the models used for data analysis are actually functions of the parameters of an underlying true model. Therefore, in order to fully understand what a proposed model actually represents, it is useful to examine how the parameters in an approximating model relate to the parameters in the true model. That is, given a statistic, this paper seeks to determine what the statistic is actually estimating in terms of an underlying true model. Examples and illustrations of the meaning of parameters in an approximating model to a true underlying model are provided. This is accomplished by fitting an approximating model to an assumed true model, similar to the way an approximating model is fit to a data set. The ideas are also illustrated with latent variable models, in particular, using mixture models.

Keywords Finite mixture; Latent variables; Least-squares; Maximum likelihood estimation; Regression.

1. Introduction

One of the most famous statistical quotes is by George Box: "all models are wrong, some are useful." This quote appears in Box and Draper [1] (page 424) where the authors go on to say that "all models are approximations ... the approximate nature of the model must always be borne in mind." From this perspective, one could argue that all models are under-specified or maybe even misspecified. Models are usually defined in terms of parameters, and the estimates of these parameters are used to gain insight and make statistical inference on the population that generated the data. However, if the models used in practice are approximations to some true underlying model, then it is useful to understand how the parameters in a proposed model relate to the parameters in an underlying model that is either the true model, or a model closer to the truth. In some cases, the parameters in a proposed approximation model may provide useful insight into the underlying population, and in other cases the parameters in an approximation model may have no useful meaning at all. It is important to distinguish these two very different scenarios. For instance, fitting a straight line to curvilinear data extracts the linear trend in the

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Parameter Estimation of a Skewed Double Exponential Distribution

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ABSTRACT Jagannathan *et al.* [3] introduced the Skewed Double Exponential (SDE) distribution and presented various stochastic representations and characterizations. This paper deals with statistical inference concerning the parameter of the SDE family of distributions. We present two estimators of the shape parameter (λ) and discuss their properties. Simulation studies are also included to provide the intuition for the results.

Keywords Method of moments, Maximum likelihood, Skewed double exponential, Asymptotic unbiasedness, Consistency of estimators.

1. Introduction

There are many methods to introduce skewness into statistical models. Azzalini [1] provides one such method in defining the Skew Normal distribution. This distribution, however, was first mentioned in Roberts [6]. Jagannathan *et al*. [3] used a similar approach to introduce the Skewed Double Exponential (SDE) distribution, defined below.

Definition 1 A random variable Y is said to have a skewed double exponential distribution with parameter (λ), denoted SDE(λ), if its density is given by

$$g(y, \lambda) = 2f(y)F(\lambda y) \tag{1}$$

where f and F are respectively the density function and the distribution function of a DE(0, 1) distribution.

Another approach to skew the DE(0, 1) model was discussed in Kozubowski and Podgórski [4] where they define the asymmetric Laplace (AL) laws. Jagannathan *et al*. [3] also discussed basic properties of the SDE family of distributions and provided stochastic representations and

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Confidence Sets with Asymptotically Constant Coverage Probability Centered at the Positive Part James-Stein Estimator

Supranee Lisawadi
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ABSTRACT Asymptotic expansions for coverage probabilities by (1− α)-confidence sets centered at James-Stein estimate show that this probability depends on noncentrality parameter τ^2 which is a sum of squares of mean values of normal distributions under confidence estimation. In the present paper we show how these asymptotic expansion can be used for a construction of a confidence region with asymptotically ($\tau \to 0$ or $\tau \to \infty$) constant coverage probability 1− α.

Keywords Confidence sets; Positive part James-Stein estimation; Multivariate normal distribution; Coverage probability; Asymptotic expansion.

1. Introduction

We consider a problem of confidence estimation of a mean vector $\theta = (\theta_1, \cdots, \theta_p)$ of *p*-dimensional normal distribution with independent coordinates having equal variances $\sigma^2 = 1$. Let $\overline{X} = (\overline{X}_1, \cdots, \overline{X}_p)$ be a vector of sample means calculated by sample of the equal size *n* of marginal distributions. Confidence set

$$D_{\overline{X}} = \left\{ \theta : n \sum_{i=1}^{p} (\theta_i - \overline{X}_i)^2 \le c^2 \right\}$$

possesses the given confidence level $1-\alpha$, if c^2 is the quantile of chi-square distribution with *p* degrees of freedom, that is, $K_p(c^2) = 1-\alpha$ where $K_p(\cdot)$ is the chi-square distribution function.

Such confidence set enjoys the minimax property, but there are other sets with the minimax property that provide a bigger coverage probability for all values of the noncentrality parameter $\tau^2 = n\|\theta\|^2$, if $p \ge 4$. In the current article we consider one of such sets

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Bayesian Life Test Planning for a Family of Life-time Distributions: Some Approximate Solutions

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ABSTRACT For the family of lifetime distributions introduced by Moore and Bilikam [3], the problem of determining the truncation number under failure-censored data (type II censoring) is considered, while dealing with Bayesian interval estimation of the p^{th} quantile. For the specific case of Weibull distribution, approximate solutions for the equations obtained by Zhang and Meeker [4] are provided for determining truncation number and prior hyperparameters. The same inferential problem is considered for Burr and Pareto distributions and failure of criterion based on a precision factor for a credibility interval is established. The problem of point estimation is also considered and it is proved that Bayes risk provides a criterion for determining the truncation number. The same criterion can be used for dealing with interval estimation problem. Consideration is given to squared-error loss function and conjugate prior distribution.

Keywords Bayesian life-tests; Type II censoring; Conjugate prior; Hyperparameters; Truncation number; Credibility interval.

1. Introduction

A lot of work has been done in the literature for life test planning through classical and Bayesian approaches. For a brief review on the literature, one may refer to the recent paper of Zhang and Meeker [4]. For failure-censored data (type II censoring), they dealt with the problem of Bayesian interval estimation of p^{th} quantile (depending upon the scale parameter $\theta = \eta^{\beta}$) of Weibull distribution

$$f(x;\eta,\beta) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right); (x,\beta,\eta) > 0 . \tag{1.1}$$

Zhang and Meeker [4] assumed the shape parameter β to be known. They considered squared-error loss function and conjugate prior distribution for θ. The reason for assuming the shape

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Efficient Designs for Constrained Categorized Mixture Experiments

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ABSTRACT In categorized mixture experiments, the mixture consists of different categories of components known as major components. Each major component is a pure component or consists of two or more minor components. Design for such mixture experiments became very large in terms of minor components. It is impracticable in real life situations to run such large designs. So far the efficient and reduced designs for such categorized constrained mixture experiments are not available in literature. In this paper we propose altogether a new, simple and systematic algorithm, for construction of design with high G efficiency when both the major and minor components have constrained boundaries.

Keywords Major components; Minor components; Mixture experiment; Categorized components; G Efficiency.

1. Introduction

The experiments which involve blending of several components are known as mixture experiments. The quality or response of the end product depends upon the relative proportion of components in a mixture. A mixture design problem is a selection of mixing proportions of q components called as mixture design points. In a mixture, component level, say, X_i satisfy the constraints,

$$0 \le l_i \le X_i \le u_i \le 1; \quad i = 1, 2, ..., q \tag{1}$$

and

$$\sum X_i = 1 \tag{2}$$

where l_i and u_i are the lower and upper specified boundaries; $i = 1, 2, \ldots, q$. Then the experi-

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Continuous Review Inventory Model for Deteriorating Items under Bulk Demand

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ABSTRACT The paper considers a continuous review inventory model for deteriorating items, when demand arrives in bulks of varying sizes. Orders are placed whenever there is depletion from stock, owing to either demand or deterioration. The lifetime of an item is assumed to be exponential so that a constant proportion of the stock on hand deteriorates per unit time. The orders are processed at a supply centre with processing time for each item having an exponential distribution. The behavior of the optimal policy and minimum expected cost with change in different unit costs is studied through numerical examples.

Keywords Deteriorating items; Bulk demand; Exponential life time; Exponential processing time; Continuous review model; One-for-one ordering policy.

1. Introduction

The traditional inventory model assumes depletion of inventory to be caused only by demand. However, when the inventory manager stocks items that deteriorate with time, loss in inventory may also occur due to deterioration. Deterioration is defined as decay, evaporation, obsolescence, loss of utility etc that results in decreasing usefulness from the original condition. Vegetables, gasoline, blood etc are examples of such products. Several authors have considered inventory models for deteriorating items. The earliest work in this line is due to Ghare and Schrader [4], who extended the standard EOQ model disallowing shortages to the case of decaying items. Their model was extended to more general types of deterioration by Covert and Philip [2], Shah [15] and Jaiswal and Shah [7], among others. Inventory for deteriorating items with constant positive lead-time was first considered by Nahamias and Wang [10], who studied

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Section of Teaching and Applications

An Elementary Central Limit Theorem

Mark Inlow
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ABSTRACT Classical central limit theorems provide conditions under which sums of random variables converge in distribution to normality. We present a central limit theorem which uses induction to prove that, for power of two sample sizes, the moments of the sample mean converge to those of the normal distribution. Since the normal distribution is determined by its moments, it follows that the sample mean is asymptotically normal. Our theorem is elementary in the sense of being completely accessible to undergraduates.

Keywords Moments; Sampling distribution; Teaching; Induction; Strong induction.

1. Introduction

Classical central limit theorems (CLT's) provide conditions under which sums of random variables converge in distribution to normality. Since the simplest CLT requires the use of moment-generating functions, little or no rigorous justification is usually provided in introductory statistics courses, in particular, post-calculus courses for engineering, mathematics, and science students. As a possible means to remedy this situation, we present a CLT accessible to students who understand moment calculations involving independent, identically distributed random variables and mathematical induction.

2. A Central Limit Theorem

Let $\{x_1,\cdots,x_n\}$ denote an independent and identically distributed (IID) sample from a zero-mean, unit-variance random variable (RV) X. Let Y_l denote the mean of a sample of size $n=2^{l-1}$ standardized to unit variance:

$$Y_l = 2^{-(l-1)/2}(x_1+\cdots+x_n). \tag{1}$$

In our CLT we prove that if the moments of X exist, then the moments of Y_l converge to those

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