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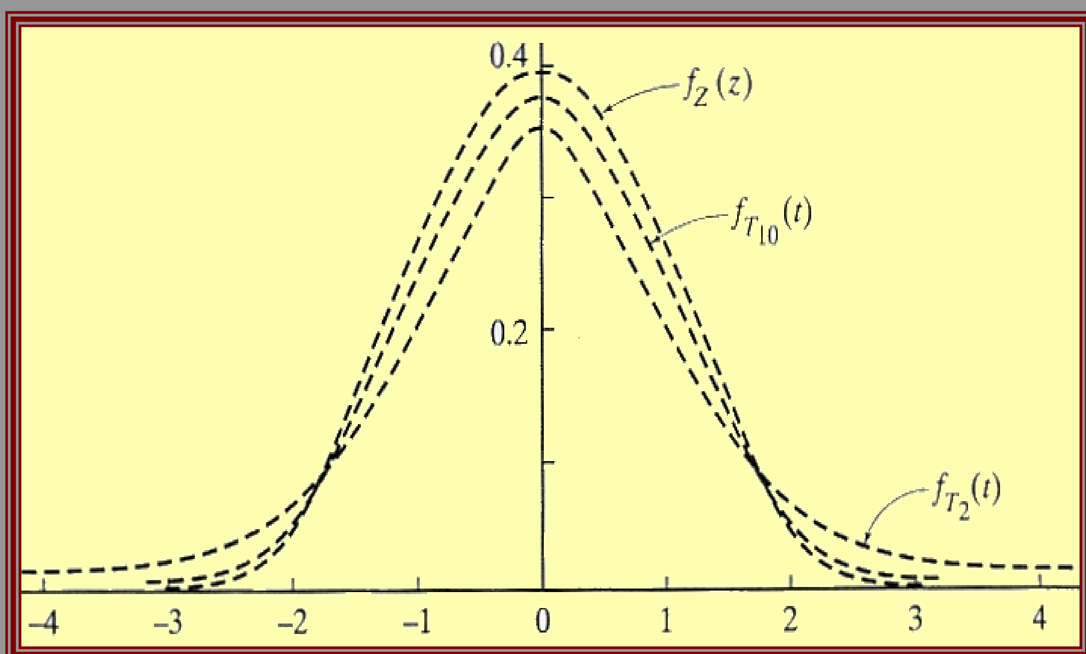
JPSS

A comprehensive journal of probability and statistics
for theorists, methodologists, practitioners, teachers, and others



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Extended Generalized Log-logistic Families of Lifetime Distributions with an Application

James U. Gleaton
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James D. Lynch
University of South Carolina

ABSTRACT The proportional odds transformation partitions the set of all lifetime distributions. In each equivalence class, members are related through proportional odds (p.o.) transformations. Each class is stochastically ordered by the transformation parameter. It is shown that either every member of a class has a m.g.f., or none does; and that the Kullback-Leibler information function for two distributions in a class is increasing in the ratio of the transformation parameters. Similar properties are shown for equivalence classes generated by an extended generalized log-logistic transformation, which is a composition of a p.o. and a log-logistic transformation. An example of data fitting is presented.

Keywords Generalized log-logistic families; Proportional odds families; Weibull distribution.

1. Introduction

In Gleaton and Lynch [5], the reliability of a particular physical system, an inhomogeneous (varying cross-sectional areas) bundle of brittle elastic fibers under a tensile load, was examined. Under the assumptions of equal load sharing and the Maximum Entropy Principle [8], it was shown that the fiber survival distributions for fibers of different cross-sections are related to each other through a generalized log-logistic transformation.

Gleaton and Lynch [6] discussed properties of lifetime distributions belonging to families generated by a generalized log-logistic (g.l.l.) transformation:

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Estimation of Pr(*Y*<*X*) When *X* and *Y* Belong to Different Distribution Families

M. Masoom Ali — *Ball State University*
Manisha Pal — *Calcutta University*
Jungsoo Woo — *Yeungnam University*

ABSTRACT In this paper, we consider estimation of the reliability function $\xi = \Pr(X < Y)$, when X and Y are independently distributed, with X following a Levy distribution, while Y is uniformly distributed or has a one-parameter exponential distribution. The maximum likelihood estimator and the UMVUE of ξ and its variance have been derived. Exact and asymptotic confidence intervals for ξ have also been suggested.

Keywords Levy distribution; Uniform distribution; Exponential distribution; Quotient distribution; Maximum likelihood estimator; UMVUE.

1. Introduction

Stress-strength reliability is one of the main tools of reliability analysis of structures. A stress-strength system fails as soon as the applied stress Y is at least as large as its strength X. This model is also known as the load-capacity model in the context of solid mechanics or structural engineering. Inference regarding $P(X < Y)$, defining the reliability of the system, has been widely discussed in literature, when X and Y are assumed to be independent random variables belonging to the same univariate family of distributions (see, for example, Pal *et al*. [7], Ali *et al*. [1] and [2]). From practical viewpoint, however, it is not unusual for X and Y to belong to different families. To the best of our knowledge, only Nandi and Aich [6] studied such a situation. They derived the maximum likelihood estimate of $P(X < Y)$, when X follows exponential distribution and Y has inverse Gaussian, half normal or half Cauchy distribution.

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Optimum Three-Step Step-Stress Plans for Cumulative Exposure Model Using Log-Logistic Distribution

Mohammed Al-Haj Ebrahem and Abedel-Qader Al-Masri
Yarmouk University

ABSTRACT This paper presents the optimum times of changing stress level for a three-step step-stress plans. A log-logistic distribution and a cumulative exposure model are assumed. Optimum times of changing stress level are obtained by minimizing, with respect to the change times, the asymptotic variance of the maximum likelihood estimator of a given 100 p-th percentile of the distribution at the design stress. Tables of optimum times for various values of the model parameters are presented. The asymptotic normality of the maximum likelihood estimator is used to construct confidence intervals of the model parameters and illustrated by an example.

Keywords Accelerated life test; Cumulative exposure model; Log-logistic distribution; Maximum likelihood estimation; Three-step step-stress.

1. Introduction

The log-logistic distribution arises in a variety of fields. The special features of this distribution together with its relation with the logistic distribution have allowed it to be used as a model in various real life applications.

In accelerated life testing, information on the life distribution of the products can be quickly obtained by testing them at higher than design level of stress. Data are obtained at accelerated conditions, and based on the relationship between the lifetime and the stress levels, results are extrapolated to the design stress. One way of applying stress to the test units is a step-stress test which allows the stress of a unit to be changed at specified times. Nelson [10] described this important type of accelerated life test. In the step-stress test, an initial low stress is applied to all test units. If a unit does not fail in a specific time, the stress is increased. There can be more than one change of stress level. If there is a single change of stress, the accelerated life test is called a simple step-stress test.

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Exact Inference for Testing Spatial Segregation by Nearest Neighbor Contingency Tables

Elvan Ceyhan
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ABSTRACT Nearest neighbor methods are widely used in the analysis of spatial point patterns in ecology and environmental sciences. We present exact inference on tests based on nearest neighbor contingency tables (i.e., NNCT-tests) for testing segregation and association patterns. The spatial pattern of *segregation* occurs when members of a class tend to be found near members of the same class (i.e., conspecifics), while *association* occurs when members of a class tend to be found near members of the other class or classes. The null hypothesis is randomness in the nearest neighbor structure, which may result from — among other patterns — *random labeling* (RL) or *complete spatial randomness* (CSR) of points from two or more classes (which is called *CSR independence*, henceforth). Pielou's, Dixon's, and various other NNCT-tests rely on asymptotic approximations. Exact inference has been extensively used on contingency tables in general, but not for NNCT-tests. We propose several variants of Fisher's exact test on NNCTs for testing CSR independence or RL with one- or two-sided alternatives, as well as variants of the exact version of Pearson's test on NNCTs for the two-sided alternative. We also perform a correction on odds ratio, the parameter used in exact inference for contingency tables. An extensive Monte Carlo study is provided for empirical significance level (i.e., Type I error rate) and power comparisons. We demonstrate that the most conservative versions of the exact tests have the appropriate level and higher power compared to other exact and asymptotic NNCT-tests.

Keywords Association; Clustering; Complete spatial randomness; Fisher's exact test; Independence; Random labeling; Spatial point pattern.

1. Introduction

Spatial point patterns have important implications in various fields such as ecology, population biology, and epidemiology. Most of the related research has been on patterns of one

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Bias-Adjusted McNemar's Test for Misclassified Data

Tze-San Lee
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ABSTRACT Although the 1:1 pair-matching case-control study subjected to the misclassified data has been studied by several authors, constraints imposed on the sensitivity and specificity parameters of the classification procedure were not adequate to guarantee that their bias-corrected odds ratio is positive. Here is presented a bias-adjusted McNemar's test with adequate constraints to remove the above mentioned deficiency. Furthermore, if a validation sample is not available, I have shown how to calculate the true classification sensitivity and specificity from the data available in the main study. Then, a sensitivity analysis is performed to study what the possible misclassification effects are. The data taken from a study on the association between the acute myocardial infarction and gallbladder patients among American Navajo Indians with respect to the etiologic factor of diabetes was used to illustrate how to calculate the true classification sensitivity and specificity. Subsequently, I conducted a sensitivity analysis for the bias-adjusted McNemar's test.

Keywords McNemar's test; Misclassification error; Pair-matching study; Sensitivity; Specificity; Validation sample.

1. Introduction

McNemar's test is used to compare the proportions between two correlated binary variables in a 1:1 pair-matching case-control study (McNemar [21]). There is a rich literature with research on the extension and application of this test. See, for example, Bennett-Underwood [2], Cochran [4], Duffy [7], Liang-Zeger [18], Mantel-Fleiss [20], Stuart [26], and Suissa-Schuster [27]. Yet, the restriction on using the original McNemar's test is that it can be used only for an "independent" sample of paired binary data. If the pairs are repeated measurements on the same subject, a straightforward application of McNemar's test is not valid. For such a case, McNemar's test has to be modified for clustered binary pairs (Eliasziw-Donner [9]; Qu-Easley [25]).

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The Overall *F*-tests for Seasonal Unit Roots under Nonstationary Alternatives: Some Theoretical Results and a Monte Carlo Investigation

Ghassen El Montasser
Manouba University

ABSTRACT In many empirical studies concerning seasonal time series, it has been shown that the whole set of unit roots associated with seasonal random walks are not present. I focussed, in this paper, on the overall *F*-tests for seasonal unit roots under some nonstationary alternatives different from the seasonal random walk. In this case, I established the asymptotic theory of these tests using a new approach based on circulant matrix concepts. The simulation results joined to this theoretic analysis showed that these overall *F*-tests as well as their augmented versions maintained high power against these nonstationary alternatives.

Keywords Kunst test; Nonstationary alternatives; Brownian motion; Monte Carlo simulation.

1. Introduction

The stochastic nature of the seasonality seems to gain ground in empirical studies. Several aspects related to seasonal unit root tests were treated in the literature. In this respect, the power of these tests against nonstationary alternatives is an important issue that recently acquired some concern. To the best of our knowledge, Ghysels, Lee, and Noh [GLN] [5] are the first authors who studied this question. In fact, in a Monte Carlo study, they showed that against a nonseasonal random walk, the power of the tests of Dickey, Hasza and Fuller [2] lies well lower than that of the tests of Hylleberg, Engle, Granger and Yoo [HEGY] [9]. Ghysels *et al*. [5] guessed that “the Dickey *et al*. [2] test may not separate unit roots at each frequency” (p. 432). The restriction behind the Dickey *et al*. [2] procedure is that all the unit roots (conventional and

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A Unified Framework for Gaussian and Non-Gaussian AR(1) Modelling

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ABSTRACT In this paper, both Gaussian and non-Gaussian time series models are brought to the same platform. First order autoregressive processes (AR(1)) with convolution of Gaussian and non-Gaussian marginals having additive structure is discussed. The role of self-decomposable and semi-self-decomposable laws with such AR(1) models is discussed. As an illustration, AR(1) processes with generalized normal semi-α-Laplace marginals are developed. Some properties of the new processes along with higher order processes are considered. Potential applications are discussed.

Keywords Autoregressive processes; Generalized normal semi-α-Laplace distribution; Non-Gaussianity; Semi-self-decomposable.

1. Introduction

Gaussianity, linearity, and stationarity were the basic assumptions of time series modeling till 1980. By the pioneering work of Gaver and Lewis [2], introducing exponential autoregressive (EAR) models, a new era of non-Gaussian time series modelling was opened up. During the same time various non-linear models were also introduced by various researchers such as Tong and Lim [23], and Rao [11] based on the evidences provided by different data sets from socio-economic contexts, hydrological and reliability contexts and many other areas. The need for non-Gaussian autoregressive models have been long felt from the fact that many naturally arising time series are clearly non-Gaussian with Markovian dependence structure. A large number of non-Gaussian time series models have been introduced and studied by various authors. Such models are found to be useful in modeling data from various contexts such as reliability, hydrology, biology, socio-economics and finance.

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An Identity for the Exponential Family Model

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ABSTRACT Stein [5] proved an identity for the normal distribution by using integration by parts. Hudson [4] extended Stein's result to the case of exponential family of distributions. In this paper, we give a generalization of Hudson's identity. Some applications of the new identity for finding the UMVUE's (uniformly minimum unbiased estimators) of some parameters are also given.

Keywords Stein's Identity; Hudson's identity; Exponential family of distributions.

1. Introduction

Stein [5] proved that if $X \sim N(\theta,1)$ and $g(x)$ is a differentiable function with $E_\theta |g'(X)| < \infty$, then

$$E_\theta[g(X)(X-\theta)] = E_\theta[g'(X)]. \tag{1.1}$$

Hudson [4] generalized (1.1) for the exponential family model where he proved that, if the p.d.f. of X is given by

$$f(x,\theta) = A(x)e^{\theta x - B(\theta)} \tag{1.2}$$

where θ ranges over the natural parameter space, $A(x)$ is a nonnegative measurable function of x which does not depend on θ and $B(\theta)$ is a function of θ only. If the support of X is R (the set of real numbers) and g is a differentiable function on R satisfying $E_\theta |g'(X)| < \infty, \forall\theta$, then

$$E_\theta\{g(X)[T(X)-\theta]\} = E_\theta[g'(X)],\ \forall\theta \tag{1.3}$$

where $T(x) = -A'(x)/A(x)$.

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On the Generalized Lagrangian Probability Distributions

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ABSTRACT In this paper, some properties of the generalized Lagrangian probability distribution are derived. We show that the generalized Lagrangian distribution of the first kind generated at different expansion points can be obtained from one another. General formulas for finding the moments are obtained. The relationship between the Lagrangian distribution and the generalized Lagrangian distribution are investigated. The equivalence theorem for the Lagrangian distribution of the first kind and the Lagrangian distribution of the second kind is extended to the generalized classes.

Keywords Lagrange expansion; Moments; Equivalence.

1. Introduction

The class of Lagrangian distributions (L), including Lagrangian distribution of the first kind (L_1) and Lagrangian distribution of the second kind (L_2), was derived from the Lagrangian expansions, which were introduced by Lagrange (1736-1813) at the end of eighteenth century. The Lagrangian distribution L_1 was first introduced and studied by Consul and his associates in early 70's (Consul and Shenton [4], [5], [6], and [7]). Let $f(z)$ and $g(z)$ be two analytic functions of z which are infinitely differentiable with respect to $z \in [-1, 1]$ such that $g(0) \neq 0$, $g(1) = 1$, $f(1) = 1$, $f(0) \geq 0$ and $\{D^{x-1}[g^x(z) f'(z)]\}_{z=0} \geq 0$ for $x \in N$, where N is the set of natural numbers and $D = \partial / \partial z$. Under the Lagrangian transformation $z = g(z)$, Consul and his associates derived the probability generating function (pgf) of Lagrangian probability distribution of the first kind, $L_1(f, g; x) = L_1$, as

$$\varphi(u) = f(z) = f(0) + \sum_{x=1}^{\infty} \frac{u^x}{x!} \{D^{x-1}[g^x(z) f'(z)]\}_{z=0}. \qquad (1)$$

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