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Table of Contents

Theory and Methods

Teaching and Applications

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4(2), 123-135, Aug. 2006

On the Weak Law of Large Numbers with Random Indices for Randomly Weighted Row Sums from Arrays of Random Elements in Banach Spaces

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ABSTRACT For randomly weighted and randomly indexed sums of the form $\sum_{j=1}^{T_n} A_{nj}V_{nj}$ where $\{A_{nj}, j \ge 1, n \ge 1\}$ is an array of random variables, $\{V_{nj}, j \ge 1, n \ge 1\}$ is an array of random elements in a real separable Banach space, and $\{T_n, n \ge 1\}$ is a sequence of positive integer-valued random variables, conditions are provided under which the general weak law of large numbers $\sum_{j=1}^{T_n} A_{nj}V_{nj} \xrightarrow{P} 0$ holds. No conditions are imposed on the joint distributions of the random indices $\{T_n, n \ge 1\}$ and no independence conditions are imposed between $\{T_n, n \ge 1\}$ and $\{A_{nj}, V_{nj}, j \ge 1, n \ge 1\}$. In the main result, it is assumed that the arrays $\{A_{nj}, j \ge 1, n \ge 1\}$ and $\{V_{nj}, j \ge 1, n \ge 1\}$ are comprised of rowwise independent random variables and random elements, respectively, the V_{nj} have mean 0, the Banach space is of Rademacher type p, and the sequences $\{A_{nj}, j \ge 1\}$ and $\{V_{nj}, j \ge 1\}$ are independent for all $n \ge 1$. The unweighted case $(A_{nj} \equiv 1)$ is also considered. Versions of the above results are also obtained without the independence, mean 0, or Rademacher type p assumptions. The sharpness of the main result is illustrated by examples.

Keywords Real separable Banach space; Rademacher type p Banach space; Array of rowwise independent random elements; Weighted sums; Random weights; Random indices; Weak law of large numbers.

1. Introduction

Consider an array $\{V_{nj}, j \ge 1, n \ge 1\}$ of random elements defined on a probability space $(\Omega, \mathcal{F}, P)$ and taking values in a real separable Banach space $\mathcal{X}$ with norm $\|\cdot\|$. Let $\{A_{nj}, j \ge 1, n \ge 1\}$

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Moments of the Correlated Bivariate Normal and *t* Distributions

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ABSTRACT Multivariate normal distribution with various forms of covariate structure are extensively used in the statistical literature. The student's *t* distribution which is closely related to the normal distribution is considered in the multiple dimensions with independent and uncorrelated marginals. In this article, we derive the complete product moments of the bivariate normal distribution. Next, we derive the complete product moments of the bivariate *t* distribution with correlated marginals that follow from multivariate *t* distribution (e.g., Anderson [1]). In addition, we obtain the incomplete and absolute product moments for the considered bivariate *t* distribution.

Keywords Bivariate normal distribution; Symmetric bivariate *t* distribution; Complete, incomplete and absolute product moment.

1. Introduction

The most important distributions in modeling and analysis of bivariate data are the bivariate normal and *t* distributions. Recently Jones [6] derived a new form of the bivariate *t* distribution assuming Z_1, Z_2 and W_1, W_2 are mutually independent random variables, where Z_i are standard normal random variables and W_i are $\chi^2_{n_i}$ random variables for i = 1, 2. He also discussed the joint distribution of the pair of random variables

$$\left(\frac{\sqrt{\nu_1}Z_1}{\sqrt{W_1}}, \frac{\sqrt{\nu_2}Z_2}{\sqrt{W_1+W_2}}\right)$$

where $\nu_1 = n_1$, $\nu_2 = n_1 + n_2$ and $\nu_1 \neq \nu_2$. His version of the bivariate *t* distribution has marginal distributions that are *t* distributions on different degrees of freedom. The spherically symmetric

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On Measures of Multivariate Concordance

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ABSTRACT In this paper we formulate a definition for measures of multivariate concordance comparable to the set of axioms proposed by Scarsini [9] for the bivariate case. In particular, we study measures of multivariate concordance based on orthant dependence, and provide their sample version. Several examples illustrate our results.

Keywords Copulas; Measures of association; Multivariate concordance; Orthant dependence.

1. Introduction

The problem of association between random variables has been largely studied in literature. The development of the theory of copulas has had a great impact in the study of nonparametric measures of association in the case of continuous variables. Many measures of bivariate association such as Kendall's tau, Spearman's rho, and the medial correlation coefficient (or Blomqvist's beta) have been studied in terms of copulas. These three measures—among other—are often called measures of (bivariate) concordance because they satisfy a set of axioms proposed by Scarsini [9]. However, Scarsini's axioms only apply to a pair of random variables. In the multivariate setting, we can find generalizations of these measures (see Joe [5], Nelsen [8], and Úbeda-Flores [12]) in the sense that the random variables tend to be all large together or all small together, but such examples do not indicate the set of axioms that a measure of multivariate concordance must satisfy.

After recalling the notion of a copula (for a complete survey, see Nelsen [7]) and some of its properties (Section 2), our purpose is to formulate a definition for measures of multivariate concordance (Section 3) comparable to the set of axioms for the bivariate case, responding to

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A General Method of Constructing Extended Families of Distributions from an Existing Continuous Class

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ABSTRACT A general method of extending a continuous class of distribution (parent class) is presented. The extended class is achieved through a set of modifiers which are chosen independently of the parent class. The nature of the extended family of distributions is determined by the modifiers and the parent density. Analytical properties of the extended family of distributions are discussed. Some examples are presented with graphical illustrations.

Keywords Beta distribution; Multi-modality; Quantiles; Skew-normal distributions; Skew-symmetric distributions.

1. Introduction

In recent years, there has been considerable interest among investigators in the construction of a general class of skewed distributions, which includes standard symmetric distributions such as the normal, $t-$, logistic and the Cauchy distributions as special cases. The underlying idea is to introduce additional parameters or parametric functions that accounts for the skewness of the distribution. Azzalini [7] defined a class of distributions, referred to as skew-normal, by introducing an additional skewness parameter, that included the normal distribution as a special case. This distribution, unlike the normal distribution, is asymmetric in general, allowing both positively and negatively skewed distributions. Azzalini and Dalla Valle [10] proposed the multivariate version of the skew-normal distribution. Statistical applications of the multivariate skew-normal distribution was considered by Azzalini and Capitanio [9]. Similar families of distributions based on other symmetric distributions were also considered such as skew-Cauchy (Arnold and Beaver [2], Wahed [14]), skew-t (Wahed [14], Branco and

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Burr Type X Distribution: Revisited

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ABSTRACT In this paper, we consider the two-parameter Burr-Type X distribution. We observe several interesting properties of this distribution. This particular skewed distribution can be used quite effectively in analyzing lifetime data. It has some interesting relations with the well studied gamma, Weibull distributions and with the recently proposed exponentiated exponential and exponentiated Weibull distributions. Statistical inferences about the shape and scale parameters have been discussed. Analysis of a real data set has been performed.

Keywords Generalized Weibull distribution; Generalized exponential distribution; Hazard function; Fisher information matrix; Order statistics.

1. Introduction

Burr [5] introduced twelve different forms of cumulative distribution functions for modelling data. Among those twelve distribution functions, Burr-Type X and Burr-Type XII received the maximum attention. There is a thorough analysis of Burr-Type XII distribution in Rodriguez [21], see also Wingo [25] for a nice account of it.

In this paper, we consider the two-parameter Burr-Type X distribution. Two-parameter Burr-Type X distribution has the following cumulative distribution function (CDF)

$$F(x;\alpha,\lambda) = (1-e^{-(\lambda x)^2})^{\alpha}; \quad x>0,\ \alpha>0,\ \lambda>0, \tag{1.1}$$

where α and λ are shape and scale parameters, respectively. Several aspects of the one-parameter ($\alpha = 1$) Burr-Type X distribution were studied by Sartawi and Abu-Salih [22], Jaheen [10] & [11], Ahmad *et al*. [1], Raqab [20], and Surles and Padgett [23]. Recently Surles

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Application of Non Zero Inflated Modified Power Series Distribution in Genetics

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ABSTRACT In this paper the relationship between the number of boys and girls in a family, between the number of animals trapped and untrapped in an animal trapping experiment, between the surviving and non-surviving eggs laid by an insect, is studied. The general expression for the correlation coefficient, ρ, is obtained when N, the family size, has a Non-Zero Inflated Modified Power Series Distribution (NZIMPSD) introduced by Murat and Szynal [9]. This is then specialized to non-zero inflated generalized negative binomial, non-zero inflated generalized Poisson and non-zero inflated generalized logarithmic series distributions.

Keywords Non-zero inflated modified power series distribution (NZIMPSD), Non-zero inflated generalized negative binomial distribution (NZIGNBD), Non-zero inflated generalized Poisson distribution (NZIGPD), Non-zero inflated generalized logarithmic series distribution (NZIGLSD), Correlation coefficient (ρ).

1. Introduction

Suppose we are interested in studying the relationship between the number of boys and girls in a family, between the number of animals trapped and untrapped in an animal trapping experiment, between the surviving and non surviving eggs laid by an insect. More generally, let X be a binomial random variable with parameters N and p and we are interested in the correlation coefficient ρ between X and $Y = N - X$. In case N is constant, it is evident that $\rho = -1$ but the problem under consideration here is to investigate the value of ρ when N itself is a random variable.

Kojima and Kelleher [8] showed that in case of boys and girls in a family, negative binomial distribution is appropriate as a distribution of the family size. Gupta [4] studied the

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Sequential Point Estimation of a Function of the Scale Parameter in a New Family of Distributions

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ABSTRACT The sequential point estimation of a function of the scale parameter θ, in a family of distribution which is called the life-time family, is considered in this paper. Also, the stopping rule N subject to the risk function $R_N = E[(\hat{\gamma} - \gamma)^2] + cE(N)$, where $c > 0$ is the known cost per unit sample, is obtained. We show that in estimating a function of the Exponential scale parameter our result coincides with that of Uno and Isogai [8] and Woodroofe [10].

Keywords Sequential estimation; Stopping rule; Regret analysis.

1. Introduction

The problem of sequential estimation refers to any estimation technique for which the total number of observations used is not a degenerate random variable. In some problems the sequential estimation must be used because no procedure using a preassigned nonrandom sample size can achieve the desired objective.

The problem of sequential analysis was first studied by Wald [9], who introduced the concept of Sequential Probability Ratio Test (SPRT). Handle [3] and Stein [7] showed how sequential methods can be used to tackle some unsolved problem in point and interval estimation. Sequential estimation of scale-parameter of Exponential and Gamma distribution has been considered by Starr and Woodroofe [6], Woodroofe [10], Gosh and Mukhopadhyay [2] and Isogai and Uno [5].

In this paper, we use the results of Woodroofe [10] and Uno and Isogai [8] to obtain a sequential point estimation of function $\gamma(\theta)$, in a life-time family under the loss function given as a sum of the squared error and a linear cost.

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MLE for Exponential Model with Changing Failure Rates Based on Two-Stage Progressively Multiply Type II Censored Samples

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ABSTRACT Multiply type II censoring is a more general censoring scheme, yet mathematically and numerically it is much more complicated. For multiply type II two-stage progressive censored data from an exponential distribution, we propose MLE along with their standard errors. However, the ML method does not admit explicit solutions. Here we present a simple approximation to the likelihood equation and derive explicit estimators as linear functions of order statistics. Finally, we present an example to illustrate the method of estimation along with their standard errors.

Keywords Exponential distribution; Multiply type II censoring; Progressive; Changing failure rates; MLE; Approximate MLE; Variance.

1. Introduction

Single stage progressively type I and type II censoring has been extensively studied by several researchers. We refer to Epstein [8], Roberts [14], Cohen and Whitten [6], Bain and Engelhardt [2], Patel and Gajjar [13] among others.

Multiply type II censoring is a generalization of type II censoring, where the number of failure r is fixed while the length of experiment time T is random. Suppose n items are placed on a life-testing experiment; instead of observing each of the first r failures, one observes only the r_1-th , r_2-th,....,r_k-th failure times $x_{r_1:n} \le \cdots \le x_{r_k:n}$ where $1 \le r_1 < r_2 < \cdots < r_k \le n$. The exact failure times of the remaining failed items in the middle are not observed. Such a sample is called multiply type II censored sample. It is particularly necessary when there is not enough time and manpower to record the failure time of each item so that only some failure times and the number of failures between them are recorded. This frequently happens in follow up studies in epidemiology , socialology, life-testing and reliability, etc. For the exponential distribution,

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Product Moments of Bivariate Wishart Distribution

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ABSTRACT The moments of the multivariate Wishart distribution is known up to the fourth order. But the product moments of the elements, namely, the two sample variances and the correlation coefficient of the bivariate Wishart distribution, are not available in general. In this paper some product moments of an arbitrary order are derived for the three elements of the bivariate Wishart distribution. A theorem containing eight identities of infinite series involving product of two gamma functions has been established to facilitate the derivation. The general nature of the theorem indicates its use in other contexts.

Keywords Bivariate Wishart distribution; Product moments, Correlation coefficient.

1. Introduction

The moments of the multivariate Wishart distribution is known up to the fourth order. Some researchers have derived moments of some special type of functions, namely, sample trace and determinant of Wishart Matrix which have had applications in estimation theory. Only a few special cases of product moments are published in the literature which are mostly considered for applications in correlation analysis. This paper aims at deriving some product moments of the three elements, namely, two sample variances and the correlation coefficient of bivariate Wishart matrix. A theorem containing eight identities of infinite series involving product of two gamma functions has been established to facilitate the derivation. This will lead to generalization of some of the published results in the area.

Let $X_1, X_2, \cdots X_N (N > p)$ be a p-dimensional independent normal random vector with mean vector $\bar{X}$ so that the sums of squares and cross product matrix is given by

$$\sum_{j=1}^{N}(X_j - \bar{X})(X_j - \bar{X})' = A\,.$$

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Section of Teaching and Applications

The Moment Generating Function of the Logistic Distribution through Residue Theory

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ABSTRACT A moment generating function (mgf) is used, as its name suggests, to generate moments of a distribution. That, however, would not be its only use for us in practice. On the other hand, it is easier in some cases to calculate moments directly than to use the mgf. Therefore, the main use of the mgf is not only to generate moments, but also to help in characterizing a distribution. This property of characterization can lead to some extremely powerful results when used properly. In this note we study a computation of the mgf for the logistic distribution through residue theory. The first four moments are generated using this mgf without too much tedious calculation. The mgf of a generalized logistic distribution is discussed as well.

Keywords Branch; Branch cut; Branch point; Closed contour; Logistic distribution; Moment generating function; Pole; Polygamma function; Residue.

1. Introduction

The logistic distribution Y is a continuous random variable with cumulative distribution function (cdf) given by

$$F(y;\alpha,\beta)=\frac{1}{1+e^{-(y-\alpha)/\beta}}, \quad -\infty<y<\infty, \tag{1.1}$$

where $-\infty<\alpha<\infty$ and $\beta>0$. As usual, here α is the location parameter and β is the scale parameter. The probability density function (pdf) of Y is

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