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Aims and Scope The ***Journal of Probability and Statistical Science*** (***JPSS***, ISSN 1726-3328) is a modified version of the ***Journal of Propagations in Probability and Statistics*** (***JPPS***, ISSN 1607-7083). ***JPSS***, like its predecessor ***JPPS***, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, and any other users of probability and/or statistics. The scope of ***JPSS*** is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of ***JPSS*** to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability and/or statistics.

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Measures of Association Based on Average Quadrant Dependence

Javad Behboodian Ali Dolati Manuel Úbeda-Flores
Shiraz University *Universidad de Almería*

ABSTRACT Let H be a joint distribution function of a pair (X, Y) of continuous random variables with marginal distribution functions F and G, respectively. It is known that the expression $H(x, y) - F(x)G(y)$ measures "local" quadrant dependence at each point (x, y) in $(-\infty, \infty)^2$. In this paper we characterize measures of association based on average quadrant dependence, i.e., the expectation with respect to another joint distribution function H' — with margins F and G — of the one-dimensional random variables $H(X, Y) - F(X)G(Y)$ and $|H(X, Y) - F(X)G(Y)|$. The construction examined here includes some well-known measures of association as particular cases.

Keywords Copula; Measures of concordance; Monotone dependence; Quadrant dependence.

1. Introduction

Let X and Y be two continuous random variables with respective distribution functions F and G, and joint distribution function H. X and Y are said *positively quadrant dependent* (PQD) if $H(x, y) - F(x)G(y) \geq 0$ for all (x, y) in $(-\infty, \infty)^2$ (Lehmann [4]), and *negatively quadrant dependent* (NQD) by reversing the sense of the inequality. So, in a sense, the expression $H(x, y) - F(x)G(y)$ measures "local" quadrant dependence at each point (x, y) in $(-\infty, \infty)^2$. Let H' be another joint distribution function whose margins are again F and G. Our purpose is to study a general class of measures of association given by

$$\alpha \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[H(x, y) - F(x)G(y) \right] \mathrm{d}H'(x, y)$$

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☐ Javad Behboodian is a Professor and Ali Dolati is a Ph.D. Student in the Department of Statistics at Shiraz University, 71454 Shiraz, Iran; emails: behboodian@stat.susc.ac.ir and dolati@yahoo.com. Manuel Úbeda-Flores (corresponding author) is an Assistant Professor in the Departamento de Estadística y Matemática Aplicada, Universidad de Almería, 04120 Almeria, Spain; email: mubeda @ual.es.

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On the Finite Buffer Bulk Service $M/G/1$ Queue with Multiple Vacations

U. C. Gupta and Karabi Sikdar
Indian Institute of Technology

ABSTRACT We consider a finite buffer bulk service queue in which the server is allowed to take repeated (multiple) vacations, and each time it has emptied the queue until it finds a customer waiting. The distributions of the number of customers in the queue at service completion, vacation termination and arbitrary epochs have been obtained. Finally, some numerical results along with key performance measures such as average queue length, probability of blocking, etc., are presented.

Keywords Bulk service; Finite buffer; Queue; Single server; Multiple vacations.

1. Introduction

Over the last decade there has been tremendous surge of interest in analyzing queues with server vacations due to their applicability to the performance modeling of many engineering systems such as computer-communication networks, and manufacturing systems etc. It is observed that in several applications customers are served by a single server in batches of variable/fixed size and server is allowed to take repeated vacations if no customers are found in the queue after service completion. This time may be utilized by the server to carry out some additional work. Most of the previous studies on vacation queues assume an infinite buffer. However, in real applications queues with finite buffer are more realistic than queues with infinite buffer.

In past, several authors have analyzed various types of queues with single (multiple) vacation(s), and there is vast amount of literature available on this topic. In particular, very few of them discussed these queues with finite buffers. The $M/G/1/N$ queue with multiple

☐ Received November 2004, revised April 2005, in final form May 2005.
☐ U. C. Gupta and Karabi Sikdar are affiliated to the Department of Mathematics at the Indian Institute of Technology, Kharagpur 721302, India. Email address of U. C. Gupta: umesh@maths.iitkgp.ernet.in.

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Testing NRBU and NRBUE Classes of Life Distributions Using U-Test

M. I. Hendi S. E. Abu-Youssef Khalaf S. Sultan
King Saud University

ABSTRACT The new renewal better than used (NRBU) and the new renewal better than used in expectation (NRBUE) classes of life distributions, which are derived from comparing random remaining life at a certain age with its equilibrium (stationary) limit as the age tends to infinity, and their dual classes; the new renewal worse than used (NRWU) and the new renewal worse than used in expectation (NRWUE), have been considered in the literature for applications such as biomedical researches, engineering and statistics. In this paper, testing exponentiality against strictly NRBU (NRBUE) alternatives, or their duals is investigated through the U-test. The percentiles of these tests are tabulated for sample sizes n = 5(1)40. The power estimates of the tests are simulated for some commonly used distributions in reliability. Pitman's asymptotic efficiency of the tests are calculated for both classes and compared. Data of 40 patients suffering from blood cancer disease (Leukemia) is considered as a practical application of the proposed tests in the medical sciences.

Keywords Equilibrium life; Life families of positive aging comparison in expectation; Stochastic comparison; U-statistic; Asymptotic normality; Power estimates asymptotic relative efficiency; Testing exponentiality.

1. Introduction

The aging life is usually characterized by a nonnegative random variable $X \geq 0$ with distribution function (cdf) F and survival function (sf) $\overline{F} = 1 - F$. Associated with X is the notion of "random remaining life" at age t, denoted by X_t, where X_t has an sf as

$$\overline{F}_t(x) = \frac{\overline{F}(x+t)}{\overline{F}(t)}, \quad x, t \geq 0. \tag{1.1}$$

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☐ Authors of this paper are all affiliated to the Department of Statistics & Operations Research, College of Science, King Saud University, Riyadh 11451, Saudi Arabia. Email address of Khalaf S. Sultan (corresponding author): ksultan@ksu.du.sa.

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Alternative Variance Estimators for Optional Randomized Response Technique of Christofides' Randomized Response Device

Amitava Saha
Directorate General of Mines Safety

ABSTRACT Christofides [6] proposed a generalization of Warner's [18] method for estimating the proportion of persons bearing a stigmatizing characteristic in a population when the samples are selected with simple random sampling with replacement (SRSWR). Chaudhuri [2] extended it to complex sample surveys and also indicated how direct responses (DR's) offered by a group of sampled individuals not considering the character sensitive enough can be accommodated in this device. Here we propose four alternative unbiased estimators for the variance of the estimator of a proportion when the option for providing DR's is permitted. A numerical study illustrating the relative performances of the alternative variance estimators using artificial data is also reported.

Keywords Compulsory randomized response; Optional randomized response; Sensitive character; Varying probability sampling.

1. Introduction

In sample surveys, information collected on sensitive or socially disapproved characteristics such as induced abortion, testing H.I.V. positive, usage of illicit drugs, child abuse, habits of drunken driving, economic fraud etc., through personal interviews often suffer from answer bias and answer refusals. Many-a-times the respondents are unwilling to disclose the honest reply through direct questions on such sensitive issues. Warner [18] proposed a device called "randomized response" (RR) with a view to reduce answer bias and refusals in sample surveys involving collection of information on stigmatizing characteristics.

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☐ Amitava Saha is a Deputy Director in the Directorate General of Mines Safety, Dhanbad, Jharkhand-82600, India; email: saha_amitava@hotmail.com.

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On Asymptotically Efficient Sequential Tests for First Order Autoregressive Time Series with a Unit Root

Richard A. Levine
San Diego State University

Debasis Bhattacharya
Visva-Bharti University

ABSTRACT We derive sequential procedures for testing the unit root hypothesis of an AR(1) process. We show that these test statistics have limiting distributions that are Gaussian under the null, unit root, hypothesis and approximately Gaussian under alternative root structures in many circumstances. Consequently, critical values and power of the tests may be easily obtained and studied giving due regard to the forms of the distributions under different alternative root structures. We compare the tests developed with competing procedures in terms of level or size (particularly unbiasedness) and power. We also illustrate the ease of application of our routines with two data sets studying velocity of money in the United States from 1869 to 1960 and annually averaged prices of one-family houses in the United States from 1863 to 1999.

Keywords Test of autoregressive parameter; Power analysis.

1. Introduction

Let the autoregressive process of order one (AR(1)), $\{X_t, t \geq 0\}$ be defined as a solution of the difference equation

$$X_t = \theta X_{t-1} + \varepsilon_t,\ t \geq 1, X_0 = 0 \quad \text{with probability 1,} \tag{1}$$

where θ is a constant known as the autoregressive parameter, $\{\varepsilon_t\}$ is a non-observable sequence of independently and identically distributed (iid) normal random variables with mean zero and standard deviation σ. It is well known that the nature of the process $\{X_t\}$

☐ Received December 2004, revised May 2005, in final form June 2005.
☐ Richard A. Levine (corresponding author) is an Associate Professor in the Department of Mathematics and Statistics at San Diego State University, San Diego, CA 92182, USA; email: ralevine@sciences.sdsu.edu. Debasis Bhattacharya is a Professor at Visva-Bharti University, Satiniketan, India; email: debasis_us@yahoo.com.
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On Approximating the Distribution of an Alternative Statistic for Detecting Lag-*k* Serial Correlation

Deepak Sanjel — *McMaster University*
Serge B. Provost, Ian B. MacNeill — *The University of Western Ontario*

ABSTRACT This paper provides several approximations to the distribution of a statistic which was proposed by De Gooijer and MacNeill [1] to test for the presence of serial correlation at a given lag in a certain linear regression model. It is shown that the moments of this statistic, once expressed as linear combinations of product moments of quadratic forms, can be evaluated by making use of a symbolic computational methodology. Density approximants that can be expressed in terms of beta density functions or Laguerre orthogonal polynomials are then proposed. A comparative study of the various approximations is carried out and an indication is given as to the convergence of the statistic to its asymptotic distribution.

Keywords Serial correlation; Moments; Laguerre polynomial; Gamma approximation; Beta approximation.

1. Introduction

De Gooijer and MacNeill [1] introduced a family of statistics for testing whether serial correlation is present at a given lag in certain stationary processes. The proposed tests are based on partial sums of lagged cross-products of ordinary least-squares regression residuals and allow for testing against more complicated alternatives. Several approximations to the distribution of one of the test statistics are presented in this paper. As borne out by a simulation study carried out by De Gooijer and MacNeill [1], the power of this statistic generally dominates that of a standard test.

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☐ Deepak Sanjel is a Postdoctoral Fellow in the Department of Mathematics and Statistics at McMaster University, Hamilton, Ontario, Canada L8S 4K1. Serge B. Provost (corresponding author) is a Professor and Ian B. MacNeill is a Professor Emeritus in the Department of Statistical & Actuarial Sciences at The University of Western Ontario, London, Ontario, Canada N6A 5B7; emails: provost@stats.uwo.ca and ibmacneill@hotmail.com.

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Moments of Gamma Order Statistics

Arjun K. Gupta
Bowling Green State University

D. G. Kabe
St. Mary's University

ABSTRACT Thomas and Moothathu [6] attempt to derive the moments of gamma order statistics by using certain recurrence relations. However, these recurrence relations turn out to be complicated variable coefficients difference equations, whose exact solutions are nearly impossible to fathom. This paper suggests the applications of Jacobians of transformations to simplify some of the integrals occurring in gamma order statistics theory.

Keywords Gamma density; Order statistics; Moments; Jacobians of transformations.

1. Introduction

Given an ordered sample of size n, $0 < x_{(1)} < \cdots < x_{(n)} < \infty$, from the gamma density

$$g(x) = \frac{\exp\{-x\}x^{p-1}}{\Gamma(p)}, \quad 0 < x < \infty, \tag{1}$$

the joint density of $x_{(1)}, \cdots, x_{(n)}$ is

$$g(x_{(1)}, \cdots, x_{(n)}) = \frac{(n!)\exp\{-(x_{(1)} + \cdots + x_{(n)})\}(x_{(1)} \cdots x_{(n)})^{p-1}}{\Gamma(p)^n},$$
$$0 < x_{(1)} < \cdots < x_{(n)} < \infty, \tag{2}$$

and hence the h-th moment of $x_{(r)}$ is

☐ Received November 2004, revised May and June 2005, in final form July 2005.
☐ Arjun K. Gupta is a Distinguished University Professor and Professor of Mathematics & Statistics at Bowling Green State University, Bowling Green, Ohio 43403, USA; email: gupta@bgnet.bgsu.edu. Professor Gupta is a Fellow of the American Statistical Association, the Institute of Statisticians, the Royal Statistical Society of England, the Ohio Academy of Science, and an elected Member of the International Statistical Institute. D. G. Kabe is retired from the Department of Mathematics and Computer Science at St. Mary's University, Halifax, NS, Canada B3H 3C3.
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Response Surface Design for Correlated Noise Variables

David C. Drain
University of Missouri-Rolla

Connie M. Borror
University of Illinois

Christine M. Anderson-Cook
Los Alamos National Laboratory

Douglas C. Montgomery
Arizona State University

ABSTRACT Response surface designs for process robustness studies and robust parameter design have historically been chosen and performed under the assumption that noise variables are uncorrelated, but this may not always be a valid assumption. This paper presents a framework for quantifying the effects of correlation among noise variables on the estimates of model parameters. Both numeric and visual assessments of design optimality are employed. Nonstandard designs with superior performance compared to traditional designs are suggested for some example cases in which noise variables are correlated and have a practically significant interaction with control variables.

Keywords Design of experiments; Response surface methods; Variance dispersion graph; Fraction of design space graph; Prediction error variance; Slope estimation variance.

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☐ David C. Drain (corresponding author) is an Assistant Professor in the Department of Mathematics and Statistics at the University of Missouri at Rolla, Rolla, Missouri 65409-0020, USA; email: draind@umr.edu. Connie M. Borror is an Assistant Professor in the Division of Management Information at the University of Illinois at Champaign-Urbana, Champaign, Illinois 61820, USA; email: cborror@uiue.edu. Christine M. Anderson-Cook is affiliated with the Statistics Group, Los Alamos National Laboratory, Los Alamos NM 87545, USA; email c-and-cook@lanl.gov. Douglas C. Montgomery is ASU Foundation Professor of Industrial Engineering at Arizona State University, Tempe, AZ 85287, USA; email: doug.montgomery@asu.edu. Professor Montgomery is a Fellow of the American Statistical Association, the American Society for Quality Control, and the Institute of Industrial Engineering. He was the Editor of the *Journal of Quality Technology* (1995-1997).

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ML Estimation in Geometric Competing Failure Model from Type-I Censored and Group Censored Samples

N. W. Patel
St. Xavier's College

M. N. Patel
Gujarat University

ABSTRACT When items on life test are subject to two independent causes of failures, the resulting failure times can be used to estimate the parameters of the failure model. Here we have considered geometric competing risk failure model with two independent causes of failures. Maximum likelihood estimation of the parameters is carried out when the data is censored at a fixed cycle of failures (say) N_1, in case of ungroup and group censored samples. Asymptotic standard errors of the estimators are obtained for both the cases. Two illustrative examples are cited for ungroup and group competing risk models, respectively.

Keywords Competing risk; Geometric failure model; Type-I censored samples; Maximum likelihood estimation.

1. Introduction

When information is desired about the reliability of a product, often an evaluation by life-test of a sample of these items can be conducted. Many papers have been concerned with estimation of parameters for various failure time distributions when only one type of cause of failure is assumed to operate. In practice however, items on test may fail by more than one cause. After an item has failed and been removed from the test a failure mode analysis is conducted to ascertain the exact cause of failure. If there are k independent causes of failures of an item and when it can fail from any one of the k independent causes of failure, then the failure model is called "competing risks model". Continuous lifetime models on similar lines are proposed and motivated by Mendenhall and Hader [5], Cox [2], Boardman and Kendell [1], David and Moeschberger [3] and Patel and Gajjar [9].

☐ Received October 2004, revised May and June 2005, in final form June 2005.
☐ Nandita W. Patel is a Lecturer in the Department of Statistics at St. Xavier's College, Navrangpura, Ahmedabad-380009, India; email: nanditafrancis@yahoo.co.in. M. N. Patel is Professor, Department of Statistics, School of Sciences, Gujarat University, Ahmedabad-380009, India; email: mnp592000@yahoo.com.

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Comparison between Han-Bancroft and Brook Methods to Determine the Optimal Significance Level for Pre-Test Estimator

B. M. Golam Kibria
Florida International University

A. K. Md. Ehsanes Saleh
Carleton University

ABSTRACT Han and Bancroft [7] proposed the max-min rule based on relative efficiency to determine the optimal level of significance for usual pre-test estimators. Brook [4] proposed the minimax regret procedure based on risk to determine the optimal critical point for F test and corresponding significance level. This paper considered these two methods and analyzed them numerically. Brook's method is conservative for fixed sample size, whereas that of Han and Bancroft is flexible. If the researchers are concerned about the smaller size of the test, they might select Brook's method. However, if they are willing to accept higher size of test and gain higher minimum guaranteed efficiency, they should select Han and Bancroft.

Keywords Dominance; Chi-square distribution; Guaranteed efficiency; Linear regression; Preliminary test; Quadratic risk.

1. Introduction

We consider the following linear regression model, $Y \quad N(X\beta, \sigma^2 I)$, where Y is an $n \times 1$ vector of observations on the dependent variable, which follows a normal distribution with fixed mean vector $X\beta$, and known variance, $\sigma^2 I$, β is a $p \times 1$ vector of unknown regression parameters, X is an $n \times p$ known design matrix of rank $p\,(n \geq p)$. We are interested

☐ Received November 2004, revised February 2005, in final form April 2005.

☐ B. M. Golam Kibria is an Assistant Professor in the Department of Statistics at Florida International University, Miami, FL 33199, USA; email: kibriag@fiu.edu. A. K. Md. Ehsanes Saleh is a Distinguished Research Professor in the Department of Mathematics and Statistics at Carleton University, Ottawa, Ontario, K1S 5B6, Canada; email: esaleh@math. carleton.ca. Professor Saleh is a Fellow of the American Statistical Association, the Institute of Mathematical Statistics, and the Royal Statistical Society of England. He is the Editor–in-Chief of *Journal of Statistical Research*.

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Sample Size Estimation for Positive Cohen's Kappa

Tze-San Lee
CDC/NCEH

ABSTRACT Cohen's kappa coefficient is a measure that expresses the degree of inter-rater agreement on a categorical scale. In epidemiology, Cohen's kappa coefficient can similarly be applied to measure the level of reliability between two diagnostic tests for detecting a disease when neither of the two tests can be regarded as a gold standard. If Cohen's kappa coefficient is desired to be greater than a certain positive fraction, the formula for minimum required sample size is shown to be a nonlinear multivariate rational function of three independent parameters: the joint probability of both tests having positive results and both the joint probabilities of exactly one of the two tests being positive. An application to two diagnostic tests for detecting tuberculosis is given to illustrate the use of Cohen's kappa coefficient.

Keywords Cohen's kappa coefficient; Diagnostic test for tuberculosis; Nonlinear multivariate rational function; Sample size determination.

1. Introduction

The idea of kappa coefficient was developed by Cohen [2] to measure, in psychology, the degree of agreement between two raters when the two raters rate each of a sample of n subjects independently, with the ratings being on a categorical scale consisting of m categories, where $m \geq 2$ is a positive integer. Similarly, Cohen's kappa coefficient can be applied in epidemiology to express the level of reliability between two diagnostic tests for detecting a disease when neither of the two tests is a gold standard. In Table 1, a 2×2 contingency table is given as a hypothetical example, where two diagnostic tests, test 1, denoted by a random variable X, and test 2, denoted by a random variable Y, are employed to screen a sample of susceptible persons. Thus, if p_{ij} denotes the true joint probability that a randomly selected person is classified, after his/her illness is diagnosed by the two tests, into the i^{th} row category and the j^{th} column category, then

☐ Received August 2004, revised October 2004/April 2005, in final form June 2005.
☐ Tze-San Lee is Mathematical Statistician, Biometry Activity, CDC/NCEH/EHHE, Mail Stop E-70, Atlanta, GA 30333, USA; email: tjl3@cdc.gov.

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Algebraic Inequalities for Measures of Dispersion

Anwar H. Joarder and A. Laradji
King Fahd University of Petroleum and Minerals

ABSTRACT Some upper and lower bounds for sample standard deviation are established in terms of sample mean, median, range, the smallest and the largest order statistics. Upper bounds for variance are also derived for odd and even sample sizes whenever the sample observations are of the same sign. They are used to find bounds for some well-known sample statistics: z-scores, coefficient of variation, coefficient of skewness and the least squares estimator of the slope parameter in the context of a simple linear regression. Statistical inference of related parameters can be improved on the basis of these fixed sample properties.

Keywords Inequalities in statistics; Sample mean; Sample median; Standard deviation; Z-score; Coefficient of variation; Coefficient of skewness; Regression parameters.

1. Introduction

Let X be a random variable with mean μ and standard deviation σ. For $0 < p < 1$, the pth quantile x_p of X is defined by $P(X \le x_p) \ge p$ and $P(X \ge x_p) \ge 1-p$ or equivalently $P(X < x_p) \le p \le P(X \le x_p)$ (Rohatji [16], p.164). For example if $p = 1/2$, then $x_p = \tilde{\mu}$, the median of the random variable X. Page and Murty ([13] and [14]) published an elementary proof of the inequality $|\tilde{\mu} - \mu| \le \sigma$. O'Cinneide [12] presented a new proof for $|\tilde{\mu} - \mu| \le \sigma$ and stated the following generalization.

Proposition 1.1 Let X be a random variable with mean μ and standard deviation σ. Then for $0 < p < 1$ and $q = 1-p$, the following inequality holds:

$$|x_p - \mu| \le \sigma \max\{\sqrt{p/q}, \sqrt{q/p}\}$$

where x_p is the pth quantile.

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☐ Anwar H. Joarder and A. Laradji are Associate Professors in the Department of Mathematical Sciences at King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia; Emails: anwarj@ kfupm.edu.sa and alaradji@kfupm.edu.sa.
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