ISSN 1726-3328

# JPSS

# Journal of Probability and Statistical Science

A Comprehensive Journal of Probability and Statistics
for Theorists, Methodologists, Practitioners, Teachers, and Others

**Volume 2 Number 2**
**August 2004**

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We are grateful to the above named scholars for their warmhearted help in sharing the tedious editorial work with us. We believe the ***JPSS*** will be further promoted by their dedicated and professional service.

Finally, we would like to take this opportunity to express our deep appreciation to the broad readers of ***JPSS*** and hope that you will continuously support this journal.

Sincerely,

*Paul C. Chiou*

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JPSS

ISSN 1726-3328

# Journal of Probability and Statistical Science

Published by: Susan Rivers' Cultural Institute, Hsinchu, Taiwan, ROC

(Cosponsored by: The International Chinese Association of Quantitative Management, Taiwan, ROC)

**Aims and Scope** The ***Journal of Probability and Statistical Science*** (***JPSS***, ISSN 1726-3328) is a modified version of the ***Journal of Propagations in Probability and Statistics*** (***JPPS***, ISSN 1607-7083). ***JPSS***, like its predecessor ***JPPS***, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, and any other users of probability and/or statistics. The scope of ***JPSS*** is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of ***JPSS*** to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability and/or statistics.

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JPSS

Journal of Probability and Statistical Science

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# Some Additive Component Skewness Models

Barry C. Arnold and Robert J. Beaver
*University of California at Riverside*

**ABSTRACT** The relationship between multiple constraint skew normal models, additive component skew normal models and the closed skew-normal family of Dominguez-Molina *et al*. [9] is explored. When component variables are non-Gaussian, distinctions between the models can be noted.

***Keywords*** Skewed distributions; Hidden truncation: Closed skew-normal; Multiple constraints; Multivariate skew-normal .

## 1. Introduction

Life is not always symmetric. The normal model at times is a poor description of observed phenomena. Skewed models, i.e., models exhibiting varying degrees of asymmetry, are a necessary component of the modeler's tool kit. In this paper we will focus on a variety of models in which skewness is introduced via additive components. The genesis of the models to be described may be found in the discussion of what are known as skewed normal distributions (Azzalini [6, 7]) or hidden truncation models, in the phrasing of Arnold and Beaver [1, 2, 3, 4]. It will be useful to begin with a review of those basic univariate models in order to develop corresponding multivariate models.

## 2. Univariate Skew-Normal Models

The family of univariate densities

$$f(x;\lambda) = 2\varphi(x)\Phi(\lambda x) \tag{2.1}$$

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☐ Received January 2004, revised March 2004, in final form May 2004.

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ISSN 1726-3328

*Journal of Probability and Statistical Science*
**2**(2), 149-166, Aug. 2004

# Stationary Distribution of Random Walks at Stopping Times

Pu Patrick Wang William J. Gray Tsun-Zee Mai
*University of Alabama*

**ABSTRACT** In this paper, we study the stationary distribution of an unbounded random walk on the line $(-\infty, \infty)$ at certain stopping times. The consecutive moves of the random walk can be either independent or dependent. The distribution is obtained in the form of probability generating function. A contour integral is used to numerically inverting the generating function. Both analytical and numerical inversion of the solution are discussed.

***Keywords*** Laplace transforms; Numerical inversion; Probability generating function.

## 1. Introduction

Consider the well-known gambler's ruin problem [7] in which the gambler bets one dollar on each game and the probability of winning a game is $p$ and losing a game is $q = 1 - p$, $0 < p < 1$. Define stopping time $N$ as the number of consecutive losses when the gambler stops playing. The question is then: what is the gambler's profit at the stopping time $N$?

If $X_n$ denotes the profit at time $n$, it is known that the underlying Markov chain $\{X_n, n \geq 0\}$ is transient for $p \neq q$ and null recurrent if $p = q = 1/2$ so that it does not have a stationary distribution as time approaches infinity. However, if one looks at some stopping times under certain statistical regularities, a stationary distribution may exist. A stopping time may be the time when a certain pattern of the outcomes has occurred. For example, a number of consecutive wins (losses, or a combination of both) have appeared.

In this paper, we first consider a continuous-time random walk on the line $(-\infty, \infty)$. That is, the times of playing games are independent, identically distributed (i.i.d.) random

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☐ Received September 2003, revised January 2004, in final form April 2004.

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ISSN 1726-3328

Journal of Probability
and Statistical Science
**2**(2), 167-185, Aug. 2004

# Steady State Analysis of an $M^X/(G_1,G_2)/1$ Queue with Restricted Admissibility and Modified Bernoulli Schedule Server Vacations

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*Institute of Advanced Study in Science and Technology*

**ABSTRACT** We consider a two stage heterogeneous service batch arrival queue with a Bernoulli schedule server vacation, where after completion of two stages of heterogeneous service in succession the server either goes for a vacation of random length with probability $r$ ( $0 \le r \le 1$) or may continue to serve the next unit, if any, with probability $(1 - r)$. In addition, concept of restricted admissibility of arriving batches is also introduced, in which not all batches are allowed to join the system at all times. Unlike the usual batch arrival queueing system our restricted admissibility policy differs during a busy period and a vacation period. In the present paper, we derive the steady state queue size distribution at a random epoch as well as at a departure point of time. Further, we obtain some important performance measures of this model. Moreover, attempts have been made to unify several classes of related batch queueing system.

***Keywords*** $M^X/(G_1,G_2)/1$ queue; Bernoulli schedule; Restricted admissibility; Queue size.

## 1. Introduction

Control of queues is one of the most significant areas of queueing theory. In the existing queueing literature, one finds some papers, e.g., see Rue and Rosenberg [33], Stidham [37],

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☐ Received July 2003, revised February 2004, in final form April 2004.

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☐ This work is supported by Department of Atomic Energy, Government of India, NBHM Project No:- 48/2/2001/R&D II/2001 .

ISSN 1726-3328

*Journal of Probability and Statistical Science*
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# Skew Weibull Distributions on the Real Line
# I: Basic Properties

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*University of Nevada*

**ABSTRACT** We study continuous distributions on the real line, whose restrictions to positive (and negative) semi-axis are Weibull laws. Skew (and symmetric) Laplace distributions are included in this class as special cases. Here, we present main properties of these laws; maximum likelihood estimation and an application from finance are discussed in a companion paper.

***Keywords*** Asymmetric Laplace law; Double exponential distribution; Double Weibull distribution; Skew normal distribution.

## 1. Introduction

Taking the $1/\alpha$ power of a standard exponential variable with the probability density function (p.d.f.) $f(x) = e^{-x}$, $x > 0$, leads to the classical Weibull distribution with the p.d.f.

$$f(x) = \alpha x^{\alpha-1} \exp\{-x^{\alpha}\}, x > 0 \tag{1}$$

This distribution is named after Waloddi Weibull, who used it to model the breaking strength of materials (see Weibull [18, 19]) as well as in other applications, including quality control and reliability (see Weibull [20]). It is now one of the most widely used probability distributions in science and engineering (see, e.g., Halinan [7], and also extensive references in Johnson *et al.* [9]). This distribution can be extended to the whole real line by symmetrization of the density (1), leading to the p.d.f.

$$f(x) = \frac{\alpha}{2} |x|^{\alpha-1} \exp\{-|x|^{\alpha}\},\ x \neq 0, \tag{2}$$

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☐ Received February 2004, revised April 2004, in final form May 2004.

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ISSN 1726-3328

Journal of Probability
and Statistical Science
**2**(2), 199-211, Aug. 2004

# Percentage Points for Testing Intraclass Correlation Structure

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**ABSTRACT** The exact null distribution of the likelihood ratio test statistic for testing compound symmetry of a $p$-variate Gaussian model has been obtained and percentage points have been computed for $p \leq 8$. The inverse Mellin transform and calculus of residues have been used to derive these results.

***Keywords***: Distribution; Test criterion; Intraclass correlation; Null moments; Inverse Mellin transformation; Residue theorem.

## 2. Introduction

Let $\boldsymbol{X}$ be a random vector of order $p$ which is distributed as multivariate normal with mean vector $\boldsymbol{\mu}$ and positive definite covariance matrix $\Sigma$. Let $H_{vc}$ denote the hypothesis of compound symmetry, i.e.,

$$H_{vc}: \Sigma = \sigma^2[(1-\rho)I_p + \rho J] \tag{1.1}$$

against general alternatives, where $\sigma^2 > 0$ and $\rho$ $(-1/(p-1) < \rho < 1)$ are unknown scalars, $J$ is a $p \times p$ matrix with each element equals to unity and $I_p$ is an identity matrix of order $p$. The likelihood ratio test statistic $\Lambda_{vc}$ can be expressed in terms of the following criterion (Wilks [15]),

$$\Lambda_{vc} = \frac{p^{Np/2}(p-1)^{N(p-1)/2}\det(A)^{N/2}}{\{\operatorname{tr}(JA)\}^{N/2}[\operatorname{tr}\{(pI_p - J)A\}]^{N(p-1)/2}}, \tag{1.2}$$

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☐ Received November 2003, revised April 2004, in final form May 2004.

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ISSN 1726-3328

*Journal of Probability*
*and Statistical Science*
**2**(2), 213-243, Aug. 2004

# On the Estimation of a Linear Time Trend Regression with a One-way Error Component Model in the Presence of Serially Correlated Errors: Part I

Chihwa Kao
*Syracuse University*

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*Clarkson University*

**ABSTRACT** In this paper, we study the limiting distributions for ordinary least squares (OLS), fixed effects (FE), first difference (FD), and generalized least squares (GLS) estimators in a linear time trend regression with a one-way error component model in the presence of serially correlated errors. We show that the FE is asymptotically equivalent to the GLS when the error term is $I(0)$; the GLS is more efficient than the FE when the error term is $I(1)$. However, a naïve GLS (NGLS) could be less efficient than the FE or FD when there is no intercept in the model and the error term is $I(1)$. The NGLS is as efficient as the GLS both for $I(0)$ and $I(1)$ error terms when there is an intercept in the model.

***Keywords***：Random effects; Fixed effects; Nonstationary.

## 1. Introduction

In panel data there are two popular ways of estimating a regression with error components, fixed effects and random effects. The fixed effects model can be estimated by ordinary least squares by conditioning on the error components, while the random effects model is usually estimated by generalized least squares unconditionally. We know that the fixed effects estimator is always consistent, however, it will be inefficient if the effects are not correlated with the regressors. Generalized least squares will be efficient and consistent if the effects are not correlated with the regressors but will be inconsistent if the effects are correlated with the regressors. One can argue that certain institutional factors or characteristics of the data

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☐ Received October 2003, revised April 2004, in final form May 2004.

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ISSN 1726-3328

Journal of Probability
and Statistical Science
**2**(2), 245-254, Aug. 2004

# Intervened Exponential Distribution: A Reliability Model

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M. N. Patel
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**ABSTRACT** This article presents a model to analyze data collected under a preventive maintenance policy which is intended to improve the reliability of the equipments. The life time $T$ of equipment here is actually a sum of two exponential random variables with different means say $\theta$ and $\rho\theta$, respectively, $\rho > 0$ being the intervention parameter. We call such a distribution the intervened exponential distribution (IED). We obtain the maximum likelihood estimators of the model parameters along with variance-covariance matrix. We propose different test procedures for the intervention parameter and the power of the test is evaluated via Monte Carlo simulation. An example to study the goodness-of-fit of the model is also given at the end of the paper.

***Keywords*** Intervention parameter; Sufficient statistic; Variance-covariance matrix; Moment estimators; Maximum likelihood estimators; Likelihood ratio test; Power of the test.

## 1. Introduction

The problem of obtaining the distribution of the sum $T = Y + Z$ of two independent random variables $Y$ and $Z$, where $Y$ follows a zero truncated power series distribution with parameter $\theta$, and $Z$ follows a power series distribution with parameter $\rho\theta$ has been obtained by Patel and Gajjar [5]. Here the parameter $\rho \geq 0$ is an intervention parameter, and the distribution of T is termed as intervened power series distribution (IPSD). The intervened Poisson distribution (IPD) due to Shanmugam [7], geometric distribution (IGD) due to Patel and Gajjar [4], and other forms of IPD due to Huang and Funk [1] are deduced as special cases

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☐ Received November 2003, revised March 2004, in final form May 2004.

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ISSN 1726-3328

Journal of Probability
and Statistical Science
**2**(2), 255-262, Aug. 2004

# A Note on Some Distributional Properties of *K*-Records

Mohammad Z. Raqab
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M. Ahsanullah
*Rider University*

**ABSTRACT** In this note we consider the model of *k*-record statistics and the corresponding *k*-record indices. We present some distributional properties of these statistics. We provide a proof of a relation given in Nevzorov [9] and generalize Theorem 8.3.1 and Theorem 8.3.2 given in Ahsanullah [1].

***Keywords*** Record statistics; *K*-record statistics; *K*-record indices; Markov chain.

## 1. Introduction

Let $X_1$, $X_2$, ... be an infinite sequence of independent and identically distributed random variables (r.v.'s) having probability density function (pdf) $f(x)$ and cumulative distribution function (cdf) $F(x)$. Chandler [4] introduced the study of record statistics and documented many of the basic properties of records.

The model of *k*-record statistics is proposed by Dziubdziela and Kopocinski [6]. These statistics are also contained in the model of generalized order statistics introduced by Kamps [7]. Record statistics can be viewed as order statistics from a sample whose size is determined by the values and the order of occurrence of the observations. Record values are used in reliability theory. Record indices and related subjects are studied in Nevzorov [8, 9], Deheuvels and Nevzorov [5], and Nevzorov and Balakrishnan [10]. Two recent books (Arnold, *et al*. [3], and Ahsanullah and Nevzorov [2]) on records have appeared and both books have covered many distributional properties of record statistics as well as some statistical inferences. Now let us define

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☐ Received November 2003, revised March 2004, in final form May 2004.

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ISSN 1726-3328

*Journal of Probability*
*and Statistical Science*
**2**(2), 263-265, Aug. 2004

# A Proof for a Continuity Property of Positive Definite Matrices

Hui-Kuang Hsieh
*University of Massachusetts*

**ABSTRACT** In deriving the moment generating function of a quadratic form, a continuity property of positive definite matrices is used in many text books on linear models, yet no formal justification for this non-trivial property is documented. To satisfy the curiosity of both instructors and students in a linear models course, a formal proof of that property and a numerical example are provided.

***Keywords*** Positive definite matrix; Multivariate normal distribution; Quadratic form; Aitken's integral.

## 1. Introduction

Let $\mathbf{x} = (x_1, \ldots, x_p)'$ be a $p$-vector, and $M$ be a $p \times p$ symmetric matrix. In proving Aitken's integral

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp(-\frac{1}{2}\mathbf{x}'M\mathbf{x})dx_1 \cdots dx_p = (2\pi)^{\frac{1}{2}p}[det(M)]^{-1/2},$$

the matrix $M$ must be positive definite in order to get a non-singular matrix $P$ with positive determinant such that $P'MP = I$, thus

$$det(P'MP) = 1, \; det(M) = [det(P)]^{-2}, \text{ and } [det(M)]^{-1/2} = det(P)$$

(see, e.g., Searle [5], formula 24, page 43).

Further, let $X \quad N_p(\mu, V)$ and $A$ again be a $p \times p$ symmetric constant matrix, where $V$ is symmetric and positive definite. To obtain the necessary and sufficient condition, namely AV being idempotent, for $X'AX$ to have a chi-square distribution, Searle ([5], theorem 2, page 57) relies on the moment generating function (m.g.f.) of $X'AX$. In deriving the m.g.f.

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☐ Received February 2004, revised April 2004, in final form May 2004.

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ISSN 1726-3328

Journal of Probability
and Statistical Science
**2**(2), 267-274, Aug. 2004

# A Direct Approach to Generate the Multivariate Geometric Distribution

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*Western Illinois University*

**ABSTRACT** In this paper, we use a very simple approach to generate the multivariate geometric distribution. Of interest, is the result that the random variables generated not only have a marginal geometric distribution, but their sum also follows a geometric distribution. Other related results including those for the negative binomial distribution are also presented.

***Keywords*** Generating; Multivariate Geometric distribution; Conditional distribution.

## 1. Introduction

Text books in statistics at the undergraduate or beginning graduate level do not usually discuss multivariate discrete distributions except for the multinomial distribution. Also, one of the simplest univariate discrete distributions introduced at a beginning level course in statistics is the geometric distribution. This distribution is usually modeled as that of the number of failures preceding the first success in successive Bernoulli trials. Students are also familiar with the negative binomial distribution as a generalization of the geometric distribution. In other familiar terms, the sum of independent and identically geometrically distributed random variables yields the negative binomial distribution. In this paper, it is found that the sum of certain correlated and geometrically distributed random variables also follows a geometric distribution rather than a negative binomial distribution. As in the case of independent and identically negative-binomially distributed random variables, the sum of correlated and negative-binomially distributed random variables does follow the negative binomial distribution.

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☐ Received November 2003, revised March 2004, in final form May 2004.

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ISSN 1726-3328

*Journal of Probability and Statistical Science*
**2**(2), Appendix, Aug. 2004

# Appendix

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# Acknowledgements

The ***JPSS*** (and the former ***JPPS***) would like to acknowledge the service of the following probabilists and/or statisticians as referees during the period August_2001 to August_2004. An asterisk indicates refereeing for more than one paper during the period.

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JPPS ISSN 1607-7083

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