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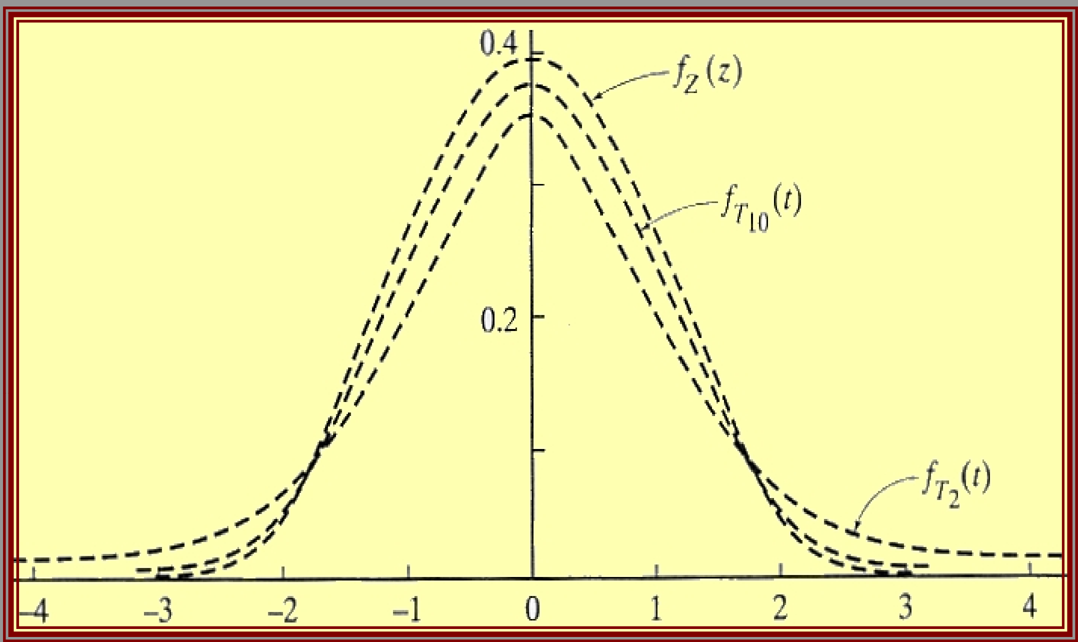
A comprehensive journal of probability and statistics
for theorists, methodologists, practitioners, teachers, and others



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A Comprehensive Journal of Probability and Statistics

for Theorists, Methodologists, Practitioners, Teachers, and Others

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Aims and Scope The ***Journal of Probability and Statistical Science*** (***JPSS***, ISSN 1726-3328) is a modified version of the ***Journal of Propagations in Probability and Statistics*** (***JPPS***, ISSN 1607-7083). ***JPSS***, like its predecessor ***JPPS***, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, students, and any other users of probability and/or statistics. The scope of ***JPSS*** is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/ or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of ***JPSS*** to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability, statistics and biostatistics.

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February 1, 2019

Editorial Note

There are eight articles on the theory, methods, teaching and applications in this issue. Among the eight articles, four are on Lindley distribution. The first article familiarizes a new distribution, called the Zografos and Balakrishnan Lindley-Poisson (ZB-LP) distribution. Some well-known distributions are special cases of this distribution. Some important properties of the proposed distribution are explored. A discrete quasi Lindley distribution, which contains Lindley distribution as a special case is introduced in article two. Some useful properties of the distribution along with parameters estimation by maximum likelihood method are given in this article. The third article derives the new stochastic representations for two-parameter Lindley distributed random variables and shows that these distributions are infinitely divisible but not geometrically infinitely divisible. A new generator with two extra parameters, namely beta odd Lindley-G family are generated in article four. Some special cases of this distribution are also presented. Some infinite series are developed to generate circular strongly partially-balanced repeated measurements designs in periods of two different sizes in article five. Article six studies the selection of optimal group size for Bayesian estimators in the cases of both binomial sampling and negative binomial sampling. To illustrate the findings, a numerical study is given in this research. There are two articles in the Section of Teaching and Applications. Article seven concentrates on the population standard deviation (PSD), and interpret it as the radius of a right circular cylinder having volume equal to that of a solid of revolution when the graph of the cumulative distribution function (CDF) is revolved about a vertical line through the mean. Finally, article eight introduces the basic concept of Markov chains to undergraduate students with backgrounds in linear algebra by giving a couple of examples showing different situations in which a Markov process reaches equilibrium.

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17(1), 1-26, Feb. 2019

A New Generalized Compound Lindley Distribution: Model, Theory and Application

Broderick O. Oluyede *Georgia Southern University*
Pinkie Melamu *Botswana International University of Science and Technology*
Adeniyi F. Fagbamigbe *Botswana International University of Science and Technology*
University of Ibadan
Boikanyo Makubate *Botswana International University of Science and Technology*

ABSTRACT A new distribution called the Zografos and Balakrishnan Lindley-Poisson (ZB-LP) distribution is introduced and its properties are explored. This distribution contains new and known sub-models, including Lindley-Poisson, Gamma Lindley and Lindley distributions. Some structural properties of the proposed ZB-LP distribution including hazard rate function, moments and conditional moments are presented. Mean deviations, Lorenz and Bonferroni curves, Rényi entropy and distribution of the order statistics, and L-moments are given. Maximum likelihood estimation technique is used to estimate the model parameters. Finally an application of the model to a real dataset is presented to illustrate the usefulness of the proposed distribution.

Keywords Gamma distribution; Generalized distribution, Lindley-Poisson distribution; Maximum likelihood estimation.

1. Introduction

Several distributions have been suggested for the modelling of lifetime and reliability data. A distribution that can be used to analyse such kind of data is the Lindley distribution. Lindley distribution was developed by Lindley [8] to demonstrate the difference between

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A Discrete Quasi Lindley Distribution

Rama Shanker   Berhane Abebe   Kamlesh Kumar Shukla
Eritrea Institute of Technology

ABSTRACT A discrete quasi Lindley distribution, a discrete analogue of quasi Lindley distribution, using infinite series approach of discretization has been introduced and shown that a discrete Lindley distribution is a particular case. Its moments and moments based descriptive statistics and generating functions have been obtained. The behavior of mean, variance, and coefficient of variation, skewness, kurtosis and index of dispersion has been discussed both numerically and graphically. Maximum likelihood estimation has been discussed for estimating the parameters of the distribution. Three examples of observed datasets have been considered for testing the goodness of fit of the proposed distribution and it shows quite satisfactory fit over Poisson-Lindley, discrete Lindley, and quasi Poisson-Lindley distributions.

Keywords Discrete quasi Lindley distribution; Discretization; Maximum likelihood estimation; Moments; Quasi Lindley distribution.

1. Introduction

The discretization of continuous distributions are gaining momentum in statistics literature for the last few decades because of the following two reasons: (i) the discrete analogue of a continuous distribution provide probability mass function (pmf) that can compete with the classical discrete distributions commonly used and (ii) the discrete analogue of a continuous distribution avoids the use of a continuous distribution in the case of strictly discrete data.

It has been observed by Lai [10] that the discretization of a continuous lifetime model is an interesting and intuitively appealing approach to derive a discrete lifetime model corresponding to the continuous one.

There are several methods available in statistical literature to derive a discrete distribution from a continuous distribution. One of the first proposed discretization methods is based

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Two-Parameter Lindley Distributions Revisited

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ABSTRACT We establish an equivalence of six families of two-parameter Lindley distributions and resolve their divisibility properties. In particular, we derive new stochastic representations for two-parameter Lindley distributed random variables and show that these distributions are infinitely divisible but not geometrically infinitely divisible. The established results allow for extending this distribution to stochastic processes with stationary and independent increments built upon this distribution.

Keywords Distribution theory; Geometric infinite divisibility; Infinite divisibility; Lindley distribution; Mixture; Pseudo-Lindley distribution; Quasi Lindley distribution; Shanker distribution; Sushila distribution.

1. Introduction

Since its introduction in Lindley [5], a probability distribution on the positive real line given by the probability density function (PDF)

$$f(x) = \frac{\theta^2}{1+\theta}(1+x)e^{-\theta x}, \quad x \in \mathbb{R}_+ \tag{1}$$

is known as Lindley distribution (LD). There is substantial and growing body of literature connected with LD, several two-parameter extensions of this model, as well as numerous "generalized" Lindley distributions, both continuous and discrete. Indeed, a quick internet search produced no fewer than six two-parameter versions of (1) and dozens of their further generalizations.

The first aim of this note is to point out that most, if not all, of the two-parameter Lindley distributions studied in the literature are actually equivalent to one another, so the actual number of various generalizations of (1) may not be as large as it appears to be. We discuss this in Section 2. The second goal is to establish a new stochastic interpretation of these two-parameter models, which in turn aids in proving their infinite divisibility. The latter property

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The Beta Odd Lindley-G Family of Distributions with Applications

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ABSTRACT We introduce a new generator with extra two parameters, namely beta odd Lindley-G family. Some special cases are presented. The new probability density function is a linear combination of the exponentiated-G density functions. Various mathematical properties of the new distribution are derived. Maximum likelihood score vector was obtained. Special case of the new generalization is applied to real data sets to evaluate the model performance.

Keywords Beta Odd Lindley-G family; Exponentiated-G distribution; Lindley distribution; Maximum likelihood score vector.

1. Introduction

Lindley [19] proposed the Lindley distribution as a counter example to fiducial statistics. This distribution proved to be more flexible than the exponential distribution in modeling hazard rate. Lindley distribution does not exhibit a constant hazard rate unlike the exponential distribution. The model works well in analyzing lifetime data especially in reliability theory (Ghitany *et al*. [14], Zakerzadeh and Dolati [39], Mazucheli and Achcar [22], Gupta and Singh

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Circular Strongly Partially-Balanced Repeated Measurements Designs in Periods of Two Different Sizes

Yashfa Nazeer Rashid Ahmed Rida Jabeen Muhammad Daniyal M. H. Tahir
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ABSTRACT Strongly balanced repeated measurements designs are useful in medicine, pharmacology, animal sciences and psychology to balance out the residual effects. When these designs require a large number of experimental subjects then strongly partially-balanced repeated measurements designs are preferred. In this article, some infinite series are developed to generate circular strongly partially-balanced repeated measurements designs in periods of two different sizes. Using cyclic shifts, construction procedures to obtain these designs for some cases are also described.

Keywords Circular strongly partially-balanced repeated measurements designs; Repeated measurements designs; Residual effects; Strongly balanced repeated measurements designs.

1. Introduction

A repeated measurements design (RMD) is strongly balanced with respect to the first-order residual effects if each treatment is immediately preceded λ times by each other treatment (including itself), Chalikias & Kounias [3]. Strongly balanced repeated measurements designs are very useful in the fields of medicine, pharmacology, animal sciences and psychology to control the residual or carry over effects. Most of the RMDs require a large number of experimental subjects to achieve the strongly balance condition which are not affordable for the experimenter (Problem 1). In these experiments, experimental subjects can be either human or animals which may die or recover during the experiments. In such cases, it becomes impossible to use the strongly balanced RMDs in equal periods sizes (Problem 2).

A RMD in which each treatment is immediately preceded λ_i times by each other treatment (including itself) then it is called strongly partially-balanced repeated measurements

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Study on Optimal Group Size in Testing

Kai Huang and Jie Mi
Florida International University

ABSTRACT In order to estimate small proportion *p* it is effcient and cost saving to apply the group testing procedures under certain conditions. The performance of the estimators, no matter frequentist estimator or Bayesian estimator, definitely depends on the group size. In the literature there are research works on the selection of optimal group size for the case of frequentist estimators. The criterion of optimality used for the selection in these researches is the asymptotic variance of the estimators. In this article we will deal with Bayesian estimators. As an alternative assessment of the accuracy of Bayesian estimators, we will introduce the Bayes asymptotic risk instead of the usual Bayes risk. For group testing procedures the closed-form expressions of the Bayes asymptotic risk can be derived and they are convex functions of group size *k*. Therefore, taking the Bayes asymptotic risk as the criterion of optimality ensures the existence and uniqueness of the optimal group size and reduce computation effort greatly. Numerical studies show that the performance of the group size selected in this way is satisfactory.

Keywords Asymptotic variance; Bayes risk; Bayesian estimator; Binomial sampling; Group testing; Negative binomial sampling; Optimal group size.

1. Introduction

Consider a population with a rare trait of interest for researchers. Denote the probability of a unit selected from the proportion at random having this trait as p. The goal is to estimate p. Usually p is very small since the trait under investigation is rare. It is known that in this case the classic way of estimating p by using sample proportion is ineffcient and not accurate. Dorfman [3] proposed group testing assuming that a group of experimental units shows the trait if and only if at least one unit in the group has the trait. This assumption is satisfied in many fields including human infectious disease, veterinary screening, drug discovery, and insect vector pathogen transmission, for individual specimen such as blood, urine, or cattle ear

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Portraying Standard Deviation via Revolution

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Mamunur Rashid *DePauw University*

ABSTRACT Standard deviation is a commonly used statistical measure that quantifies in a single number the amount of variation inherent in a set of numbers or in a random variable. We focus on the population standard deviation (PSD), and interpret it as the radius of a right circular cylinder having volume equal to that of a solid of revolution when the graph of the cumulative distribution function (CDF) is revolved about a vertical line through the mean. We hope this interpretation will refine the intuition of students and users of statistics. Once this concept of the PSD is grasped, developing an intuition for the more commonplace sample standard deviation (SSD) is straightforward.

Keywords Center; Cumulative distribution function; Dot plot; Histogram; Mean; Solid of revolution; Stick plot; Teaching statistics; Variance.

1. Introduction

The mean is a common measure of center and is a basic concept in all quantitative disciplines. It occurs frequently in many everyday life applications (Pollatsek *et al*. [6]; Lesser *et al*. [3]). The (arithmetic) mean (denoted by $\bar{x}$) is defined as the sum of the values divided by the number of values in a list, or

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i. \quad (1)$$

Students in high school — now-a-days even as early as in middle school (National Governors Association Center for Best Practices, Council of Chief State School Officers [5]) — are exposed to the notion of sample standard deviation (SSD), denoted by s, of a set of n numbers defined by

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - n\,\bar{x}^2\right]}. \quad (2)$$

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A Classroom Lesson on Transition Matrices of Markov Chains

Kuang-Chao Chang
Fu Jen Catholic University

Lie-Jane Kao
KaiNan University

ABSTRACT In probability theory, given the transition matrix and the initial state vector of a Markov chain, its state vectors may or may not converge to a limiting vector as the number of observation periods increases. In other words, a Markov process may or may not reach equilibrium. In this article, we introduce the basic concept of Markov chains to undergraduate students with backgrounds in linear algebra by giving a couple of examples showing different situations in which a Markov process reaches equilibrium. In particular, we employ the method of diagonalization of matrices to examine whether or not a Markov process reaches equilibrium.

Keywords Diagonalization of matrices; Eigenvalue; Equilibrium; Markov chains; Markov processes; Regular transition matrix; State vector; Steady-state vector; Transition matrix.

1. Introduction

In probability theory, given the transition matrix and the initial state vector of a *Markov chain*, its state vectors may or may not converge to a limiting vector as the number of observation periods increases. In other words, a *Markov process* may or may not reach *equilibrium*. In this article, we introduce the basic concept of Markov chains to undergraduate students with backgrounds in elementary linear algebra by giving a couple of examples showing different situations in which a Markov process reaches equilibrium. To make students easily and better understand how a Markov process can reach equilibrium, we employ the method of diagonalization of matrices, if possible, in those examples with detailed computations.

This article is organized as follows. In Section 2, we introduce the concept of Markov chains by giving some definitions and useful theorems. In Section 3, we give a brief review on

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