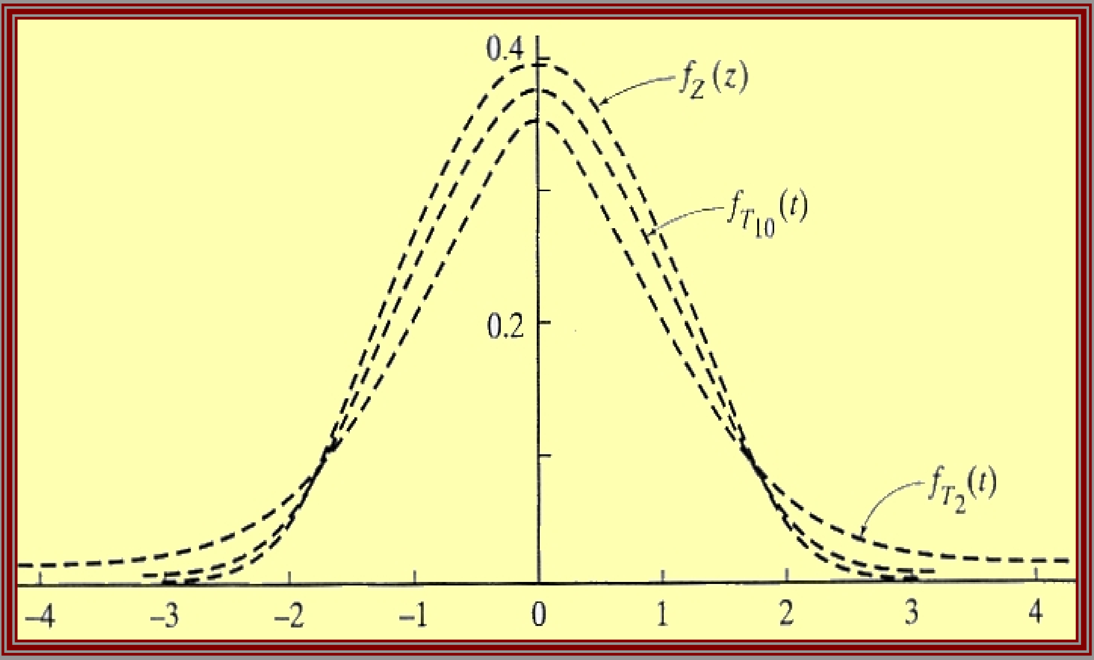
ISSN 1726-3328

# JPSS

A comprehensive journal of probability and statistics
for theorists, methodologists, practitioners, teachers, and others



# JOURNAL OF PROBABILITY AND STATISTICAL SCIENCE

Volume 15 Number 1 February 2017

**ISSN 1726-3328**

# JPSS

# Journal of Probability and Statistical Science

A Comprehensive Journal of Probability and Statistics
for Theorists, Methodologists, Practitioners, Teachers, and Others

Editor-in-Chief: **B. M. Golam Kibria**

**Volume 15 Number 1**
**February 2017**

Published by: Susan Rivers' Cultural Institute, Hsinchu City, Taiwan, ROC.

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機統學誌 JPSS ISSN 1726-3328

# Journal of Probability and Statistical Science

Published by: Susan Rivers' Cultural Institute, Hsinchu City, Taiwan, ROC

**Aims and Scope** The ***Journal of Probability and Statistical Science*** (***JPSS***, ISSN 1726-3328) is a modified version of the ***Journal of Propagations in Probability and Statistics*** (***JPPS***, ISSN 1607-7083). ***JPSS***, like its predecessor ***JPPS***, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, and any other users of probability and/or statistics. The scope of ***JPSS*** is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of ***JPSS*** to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability, statistics and biostatistics.

## Submission and Review Policies

1. Submission of manuscript written in English should be mailed (by email attachment) to the **Editor-in-Chief, an Editor, a Coordinating Editor,** or to the **Managing Editor** at the address provided in the ***JPSS*** Editorial Board.
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魏蘇珊文教事業機構發行
總公司: 中華民國臺灣新竹市建美路 2 巷 26 號。

**February 1, 2017**

# Editorial Note

There are nine articles including Editor's invited paper on the theory and methods in this issue. Most of them are on the distributional theories and their properties along with applications. The first article discusses the estimation of optimal maintenance policy in reliability. The existence and uniqueness of the MLEs and consistency of the estimators are discussed. The second article proposes a new distribution, named Poisson pseudo Lindley distribution. Different distributional characteristics of the proposed distribution along with a real life application are given in this article. A logistic-Lomax distribution and its various properties and applications are discussed in article three. Article four establishes some stochastic comparison results about sample ranges arising from exponential or PHR models. Based on generalized order statistics (gos), the Bayes and the maximum likelihood (ML) estimators have been obtained for parameters, reliability and hazard function from the two-parameter Gompertz distribution in article five. Some numerical results are presented to illustrate the performance of the procedures. An exponential method for estimating the population mean in successive sampling is discuss in article six, the properties of the suggested estimators have been investigated. Optimum replacement policy and the efficiency of the suggested estimator are given. An empirical study is carried out to demonstrate the benefit of the proposed estimator. A misclassified size-biased Poisson-Lindley distribution (MSBPLD) is defined. The method of moments and method of maximum likelihood (ML) estimation for the parameters of MSBPL distribution are investigated. in article seven, A simulation study and an application of the model to a real data set are also given. Article eight proposed a nonparametric method to estimate the time-to-failure distribution and its percentiles by using the double kernel estimator. The performance of the double kernel estimator is compared via simulation study and by using the mean square error and the length of the 95% bootstrap confidence interval as the basis criteria of the comparison. The last but not the least, article nine considers the problem of estimating the variance of a finite population, when one has already estimated the population mean by stratified random sampling. It also proposes an unbiased estimator which incorporates the known stratum weights and the mean estimator. The contents of this article may be used as supplementary materials for teachers of statistics in teaching mathematical statistics and/or survey sampling.

- **B. M. Golam Kibria**, *JPSS* Editor-in-Chief
Professor, Department of Mathematics & Statistics, Florida International University
Miami, FL 33199, USA

**ISSN 1726-3328**

# JPSS

## Journal of Probability and Statistical Science

**Volume 15 Number 1 February 2017**

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Published by: Susan Rivers' Cultural Institute, Hsinchu City, Taiwan, ROC.

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**15**(1), 1-18, Feb. 2017

**Editor's Invited Paper**

# Estimation of Optimal Policy in Reliability

Kai Huang and Jie Mi
*Florida International University*

**ABSTRACT** In reliability lots of cost functions and system performance characteristics corresponding to various maintenance policies have been derived. There is a great deal of studies on these cost functions and reliability characteristics. However, not much has been done regarding the estimation of the optimal maintenance policies. This paper considers the unit expected cost of age replacement policy and steady-state system availability. The existence and uniqueness of the MLEs of the optimal policies are established. The consistency of the estimators are also proved.

***Keywords*** Age replacement policy; Bathtub shaped failure rate; Burn-in; Cost function; Mean residual life; MLE; Steady-state system availability.

## 1. Introduction

It is certainly desirable that a practical system can work for long time without failure. There are many criteria that can be used for measuring system performance such as mean life $\mathbb{E}(X)$, reliability (survival probability) $\bar{F}(t) \equiv \mathbb{P}(X > t)$, mean residual life $\int_t^\infty \bar{F}(x)dx / \bar{F}(t)$ etc., where $X$ is the random life of system and $F(x)$ is the distribution of $X$. One way to improve the system performance after it has been produced is to use the burn-in procedure. This procedure puts systems into operation for a predetermined time, say b time units, in similar or even more severe environment than that in the field operation. Only those systems which survive the burn-in time will be shipped to vendors for field operation. A natural question is how long the burn-in time $b$ should be in order to optimize a certain reliability characteristic mentioned above since too long or too short burn-in time may even weaken the target reliability characteristic. We refer the readers to Jensen and Petersen [10], Kuo and Kuo [13], Mi [18], and Block *et al.* ([4], [5]) and references therein for more information about burn-in and its applications.

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☐Received February 2016, revised September 2016, in final form December 2016.

☐Kai Huang is an Associate Professor and Jie Mi is a Professor in the Department of Mathematics and Statistics at Florida International University; Miami, FL 33199, USA; emails: khuang@fiu.edu and mi@fiu.edu.

*JPSS*: ISSN 1726-3328

*Journal of Probability*
*and Statistical Science*
**15**(1), 19-28, Feb. 2017

# On Poisson Pseudo Lindley Distribution: Properties and Applications

Halim Zeghdoudi and Sihem Nedjar
*Badji-Mokhtar University*

**ABSTRACT** In this paper, we give a treatment of the mathematical properties for a new distribution named a Poisson pseudo Lindley distribution (PPsLD) by compounding Poisson and pseuso Lindley distributions. The properties studied include: moments, Lorenz curve, the quantile function, maximum likelihood estimation. Simulations studies and data driven applications are also reported.

***Keywords*** Lindley distribution; Maximum likelihood estimation; Poisson distribution; Pseudo distribution.

## 1. Introduction

Statistical distributions (Lifetime distributions) are commonly applied to describe real world phenomena and are most frequently used in different fields such as medicine, finance, biological engineering sciences and actuarial science. Recently, one parameter Lindley distribution has attracted the researchers for its use in modelling lifetime data, and it has been observed in several papers that this distribution has performed excellently. Let $X$ be a random variable following the one-parameter distribution with the density function

$$f(x;\theta)=\begin{cases}\dfrac{\theta^2(1+x)e^{-\theta x}}{1+\theta}, & x,\theta>0\\ 0, & \text{otherwise}\end{cases} \tag{1.1}$$

introduced by Lindley [5]. This distribution has attracted the interest of many researchers and has been generalized several times by various authors. First, [7] used (1.1) when the parameter follows a Poisson Law to derive their discrete Poisson Lindley Distribution (PLD) with density function

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☐Received May 2016, revised November 2016, in final form December 2016.

☐Halim Zeghdoudi and Sihem Nedjar are affiliated to LaPS laboratory, Badji-Mokhtar University, Annaba, 23000, Algeria; Email address of Halim Zeghdoudi: halim.zeghdoudi@univ-annaba.dz.

☐Mathematics Subject Classification: 60E05; 62H10.

*JPSS*: ISSN 1726-3328

*Journal of Probability*
*and Statistical Science*
**15**(1), 29-46, Feb. 2017

# A Study of Logistic-Lomax Distribution and Its Applications

M. Zubair / *Government Degree College Khairpur Tamewali*
*The Islamia University of Bahawalpur*
Gauss M. Cordeiro / *Federal University of Pernambuco*
M. H. Tahir / *The Islamia University of Bahawalpur*
Madiha Mahmood / *The Islamia University of Bahawalpur*
M. Mansoor / *The Islamia University of Bahawalpur*

**ABSTRACT** We study a new model called the *logistic-Lomax* distribution, which belongs to the *logistic-X* family recently proposed by Tahir *et al*. [10]. Its density function can be right-skewed, left-skewed, approximately symmetric and reversed-J shaped, and its hazard rate can have decreasing and upside-down bathtub shaped forms. Various of its structural properties are investigated including explicit expressions for the quantile function, ordinary and incomplete moments, mean deviations, generating function, a quantile power series and Shannon entropy. We discuss the estimation of the parameters by maximum likelihood and minimum spacing distance and provide some simulation results. The usefulness of the new model is proved empirically by means of two real lifetime applications.

***Keywords*** Logistic distribution; Logistic-X family; Lomax distribution; Minimum spacing distance estimator; Shannon entropy; T-X family.

## 1. Introduction

The modelling of real life phenomenons has always been attempted through proposing new generalized distributions. It has been observed that the statistical distributions represent very useful tool for describing, interpreting and predicting most of the real phenomenons. Many

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☐Received February 2016, revised July 2016, in final form August 2016.

☐M. Zubair is affiliated to the Department of Statistics at Government Degree College Khairpur Tamewali, Pakistan and the Department of Statistics at The Islamia University of Bahawalpur, Bahawalpur-63100, Pakistan. Gauss M. Cordeiro is affiliated to the Department of Statistics at Federal University of Pernambuco, Recife, PE, Brazil. M. H. Tahir (corresponding author, email: mtahir.stat@gmail.com), Madiha Mahmood, and M. Mansoor are affiliated to the Department of Statistics at The Islamia University of Bahawalpur, Bahawalpur-63100, Pakistan.

☐2000 Mathematics Subject Classification: 60E05, 62E10, 62N05.

*JPSS*: ISSN 1726-3328

*Journal of Probability*
*and Statistical Science*
**15**(1), 47-55, Feb. 2017

# On Reversed Hazard Rate Ordering of Sample Ranges from Exponential or PHR Models

Jiantian Wang
*Kean University*

**ABSTRACT** This paper establishes some stochastic comparison results about sample ranges arising from exponential or PHR models. We introduce a new partial order and show that this partial order between the two hazard rate vectors implies the reversed hazard rate order between the two sample ranges. This result extends several existing ones in the literature.

***Keywords*** Majorization; Reversed hazard rate order; Sample range; Stochastic comparison.

## 1. Introduction

Let $X_1, \cdots, X_n$ be independent random variables, and $X_{1:n} \le X_{2:n} \le \cdots \le X_{n:n}$ be the order statistics arising from $X_1, \cdots, X_n$. The statistic $X_{n:n} - X_{1:n}$ is refereed to as sample range. It is well known that $X_{k:n}$ is the lifetime of a $(n-k+1)$-out-of-$n$ system, and typically, $X_{1:n}$ and $X_{n:n}$ correspond the lifetimes of series and parallel systems, respectively. As mentioned in Ding *et al.* [2], order statistics and sample ranges play important roles in many areas of probability and statistics, such as goodness-of-fit tests, reliability theory, auction theory, actuarial science, life testing, operation research, and information sciences.

In the lifetime-related studies, exponential distribution has been widely applied because of its nice mathematical form and its unique memoryless property. For a random variable $X$ which follows an exponential distribution with hazard rate $\lambda$, we simply denote it as $X \sim \exp(\lambda)$. The topic of stochastic comparison of sample ranges arising from exponential samples has attracted considerable attention. For instance, Kochar and Rojo [3] showed that, when $X_i \sim \exp(\lambda_i)$ and $Y_i \sim \exp(\gamma_i)$, $i = 1, 2, \cdots, n,$ then

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☐Received July 2015, revised May 2016, in final form June 2016.

☐Jiantian Wang is affiliated to the School of Mathematical Sciences at Kean University, Union, NJ 07083, USA; email: jwang@kean.edu.

***JPSS***: ISSN 1726-3328

*Journal of Probability*
*and Statistical Science*
**15**(1), 57-78, Feb. 2017

# Inference and Bayesian Prediction for Gompertz Distribution under Generalized Order Statistics

M. M. Mohie El-Din　M. S. Kotb　H. A. Newer
*Al-Azhar University*　*Ain-Shams University*

**ABSTRACT** Based on generalized order statistics (gos), the Bayes and the maximum likelihood (ML) estimators have been obtained for parameters, reliability and hazard functions for the two-parameter Gompertz distribution. The symmetric (squared error loss (SEL)) and asymmetric loss functions (linear-exponential (LINEX)) are considered for Bayesian estimation. The Bayes estimators of the unknown parameters can not be obtained in closed-form and so we propose to apply Soland's method and Markov Chain Monte Carlo (MCMC) method to tackle this problem. The Bayesian prediction intervals for gos based on Gompertz distribution are obtained in one sample case. Finally, some numerical results are presented to illustrate the performance of the procedures.

***Keywords*** Bayesian estimation and prediction; Generalized order statistics; Markov chain Monte Carlo; Maximum likelihood estimates; Symmetric and asymmetric loss functions.

## 1. Introduction

The Gompertz distribution has been used as a growth model, so it's one of the most popular widely in epidemiological and biomedical studies. The Gompertz distribution was first introduced in the literature by Gompertz [10]. The probability density function (pdf) of the two parameters Gompertz distribution (denoted as $\text{Gom}(\beta,\lambda)$) is given by

$$f(x;\beta,\lambda)=\beta\exp\left\{\lambda x-\frac{\beta}{\lambda}(e^{\lambda x}-1)\right\},\ x\geq 0, \tag{1}$$

and cumulative distribution function (cdf) is

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☐Received April 2014, revised July 2016, in final form August 2016.

☐M. M. Mohie El-Din and M. S. Kotb (email: msakotb1712@yahoo.com.) are affiliated to the Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City, Cairo 11884, Egypt. H. A. Newer is affiliated to the Department of Mathematics, Faculty of Education, Ain-Shams University, Nasr City, Cairo 11757, Egypt.

***JPSS***: ISSN 1726-3328

*Journal of Probability*
*and Statistical Science*
**15**(1), 79-93, Feb. 2017

# An Exponential Method for Estimating the Population Mean in Successive Sampling

Housila P. Singh and Surya K. Pal
*Vikram University, Ujjain*

**ABSTRACT** In successive sampling, the use of auxiliary information for estimating the population mean at the current occasion is a well established fact. This paper utilizes the information on an auxiliary variable readily available on both the occasions along with the information on the study variable from the previous occasion and the current occasion. Resulting we have suggested exponential type estimators based on transformed auxiliary variable for estimating the population mean on the current occasion in two occasion successive sampling. A large number of estimators are shown as members of the suggested generalized estimator. The properties of the suggested estimator have been investigated. Optimum replacement policy and the efficiency of the suggested estimator have been worked out. An empirical study is carried out to demonstrate the merits of the proposed estimator.

***Keywords*** Auxiliary variable; Efficiency; Optimum replacement policy; Study variable; Successive sampling.

## 1. Introduction

Surveys often get repeated on several occasions for estimating same characteristics at various points of time. In successive (rotation) sampling, it is common to use the entire information gathered on the previous occasions to improve the precision of the estimators on the current occasion. It also provides the effective (in terms of cost and precision) estimates of the patterns of change over the period of time. The theory of successive sampling with a partial replacement of the sampling units was first given by Jessen [5] for analyzing the from survey data. He pioneered using the whole information gathered in the previous occasions.

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☐Received April 2016, revised July 2016, in final form September 2016.

☐Housila P. Singh and Surya K. Pal (corresponding author, email: suryakantpal6676@gmail.com) are affiliated to the School of Studies in Statistics at Vikram University, Ujjain-456010, India.

☐ASM Classification: 62D05.

***JPSS***: ISSN 1726-3328

*Journal of Probability and Statistical Science*
**15**(1), 95-112, Feb. 2017

# Classical Estimation for the Parameters of Misclassified Size-Biased Poisson Lindley Distribution and Its Applications

B. S. Trivedi
*Ahmedabad University*

M. N. Patel
*Gujarat University*

**ABSTRACT** A misclassified size-biased Poisson-Lindley distribution (MSBPLD) where some of the observations corresponding to $x = C + 1$ are misclassified as $x = C$ with probability $\alpha$, is defined. The method of moments and method of maximum likelihood (ML) estimation for the parameters of MSBPL distribution are investigated. The asymptotic variance-covariance matrices of the moment and ML estimators are obtained. We also show that these two estimators are consistent and asymptotically normal (CAN). A simulation study is presented to compare the two estimators. An application of the model to a real data set is given.

***Keywords*** CAN estimator; Maximum likelihood; Method of moments; Misclassification; Poisson Lindley distribution,; Size-biased.

## 1. Introduction

In probability theory and statistics, the Poisson distribution named after French Mathematician 'Simeon Denis Poisson' is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time and /or space if these events occur with a known average rate and independently of time since the last events. So the Poisson distribution can be applied to systems with a large number of possible events, each of which is rare. A practical application of this distribution was made by Ladislaus Bortkiewicz in 1898 when he was given the task of investigating the number of soldiers in the Prussian army killed accidently by horse kicks; this experiment introduced the Poisson distribution to the field of reliability engineering.

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☐Received October 2015, revised June 2016, in final form July 2016.

☐B. S. Trivedi is affiliated to the H. L. Institute of Commerce at Ahmedabad University, Navrangpura, Ahmedabad, India; email: bhaktida.trivedi@ahduni.edu.in. M. N. Patel is affiliated to the Department of Statistics at Gujarat University, Ahmedabad, India; email: mnpatel.stat @gmail.com.

☐MSC 2010: 62E15, 62F10, 60E05.

*JPSS*: ISSN 1726-3328

*Journal of Probability and Statistical Science*
**15**(1), 113-126, Feb. 2017

# Estimating the Time-to-Failure Distribution and Its Percentiles for Simple Linear Degradation Model Using Double Kernel Method

Omar Eidous Mohammed Al-Haj Ebrahem Laila Naji Ba Dakhn
*Yarmouk University*

**ABSTRACT** In this article, we proposed a nonparametric method to estimate the time-to-failure distribution and its percentiles by using the double kernel estimator. The performance of the double kernel estimator is compared via simulation study with maximum likelihood (ML) estimator and ordinary least square (OLS) estimator by using the mean square error and the length of the 95% bootstrap confidence interval as the basis criteria of the comparison. An application to real data set is also given. In general, the simulation results show that if the distribution random effect is chosen correctly then the ML estimator perform the best; otherwise the double kernel estimator perform the best.

***Keywords*** Classical kernel estimator; Degradation model; Double kernel estimator; Maximum likelihood estimator; Ordinary least square estimator.

## 1. Introduction

Traditional life tests are often not efficient way to obtain reliability information because few failure times data are observed by the end of the test. It is difficult to use the traditional reliability analysis that record only failure time data to analyze life time data. Thus, it is possible to get failure data by degradation measurements over time which may contain useful data about the product reliability. Most reliability studies are measured degradation as a function of time *T*. We are interested in studying the simple linear degradation model for estimating the $100r^{\text{th}}$ percentile of the time-to-failure distribution. Gertsbackh and Kordonskiy [7] discussed the degradation problem from an engineering point of view. They presented the Bernstein distribution, which describes the time-to-failure distribution for a simple linear model with random intercept and random slope. Lu *et al*. [9] compared the degradation analysis and traditional failure time

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☐Received March 2016, revised November 2016, in final form December 2016.

☐Authors of this paper are affiliated to the Department of Statistics at Yarmouk University, Irbid, Jordan; E-address of Mohammed Al-Haj Ebrahem (corresponding author): m_hassanb@hotmail.com.

***JPSS***: ISSN 1726-3328

*Journal of Probability and Statistical Science*
**15**(1), 127-133, Feb. 2017

# A Note on Stratified Random Sampling

Kuang-Min Chang
*Rutgers School of Dental Medicine*

Kuang-Chao Chang
*Fu Jen Catholic University*

**ABSTRACT** In the theory of survey sampling, stratified random sampling is often used for the purpose of estimating the population mean efficiently. After the population mean has been estimated via stratified random sampling, we may sometimes go further to estimate the variance of the population by suitably utilizing the stratified random sample and the mean estimator we already obtained. In this research, we consider such problem of estimating the variance of a finite population, when we have already estimated the population mean by stratified random sampling, and we propose an unbiased estimator which incorporates the known stratum weights and the mean estimator we already obtained. The contents of this article may be used as supplementary materials for teachers of statistics in teaching mathematical statistics and/or survey sampling.

***Keywords*** Simple random sampling ; Stratified random sampling ; Unbiased estimator.

## 1. Introduction

In the theory of survey sampling, we often want to estimate the mean of a finite population, and the simplest as well as the most commonly used sampling method is the "simple random sampling". Another useful and commonly used sampling method for estimating the mean of a finite population is the "stratified random sampling". In general, stratified random sampling is more efficient than simple random sampling for estimating the mean of a finite population if the stratification is properly used. In addition to estimating the mean of a finite population, we may also once in a while like to estimate the variance of the same finite population. For instance, in a nationwide survey research on individual's yearly income, we may want to estimate both the mean and the variance of yearly incomes of all individuals in a country. In particular, if we have applied stratified random sampling for estimating the mean of a finite population, we can go further to efficiently estimate the variance of

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□Received August 2016, revised December 2016, in final form January 2017.

□Kuang-Min Chang (BDS, DMD, and Ph.D) is an Associate Professor in the Department of Periodontics at Rutgers School of Dental Medicine, Rutgers University, 110 Bergen St., Newark, NJ 07103-2400, USA; email: dugongbu@gmail.com. Kuang-Chao Chang is a Professor in the Department of Statistics and Information Science at Fu Jen Catholic University, Hsinchuang, New Taipei City, Taiwan, ROC; email: stat1016@mail.fju.edu.tw.

*JPSS*: ISSN 1726-3328