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**Aims and Scope** The *Journal of Probability and Statistical Science (JPSS)*, ISSN 1726-3328) is a modified version of the *Journal of Propagations in Probability and Statistics (JPPS)*, ISSN 1607-7083). *JPSS*, like its predecessor *JPPS*, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, and any other users of probability and/or statistics. The scope of *JPSS* is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of *JPSS* to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability, statistics and biostatistics.

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There are nine articles on the theory and methods in this issue. Most of them are on the distributional theories and their properties along with applications. The first article contains the solution of an integral equation arising from the ruin probability of long-term bonus-malus systems. The second article proposes a new multi-parameter class of distributions called McDonald Log logistic (McLLoG) distribution, which contains several distributions such as beta Log-logistic, exponentiated Log-logistic and Log-logistic as special cases. This paper discusses different distributional characteristics of the proposed distribution and provides a real life applications. Even both generalized exponential and gamma distributions are well known for analyzing skewed life time data, however, there are some differences between them and discriminating between these two distributions are discussed in article three. A new family of the two-sided crack distribution and its theoretical properties are established in article four. Article five develops a new generalized distribution called the exponentiated log-logistic Weibull (ELLoGW) distribution, which contains exponentiated log-logistic Rayleigh, exponentiated log-logistic exponential, log-logistic Weibull, log-logistic Rayleigh, log-logistic exponential, log-logistic, Weibull, Rayleigh, exponential and several other distributions as special cases. Article six discusses some compounded poisson distributions with applications to the pattern of number of child deaths experienced by the females. Two-sided length biased inverse Gaussian distribution along with some theoretical properties and applications are discussed in article seven. Article eight compares three pairs of different types of double prior distribution for the parameter of Rayleigh lifetime model under type-II censoring and compare them with single prior distribution. A real life data are analyzed to illustrate the findings of the paper. The last but not the least, article nine contains the improved statistical inference for three-parameter crack lifetime distribution. The maximum likelihood estimation for the three parameters are developed and compared with the Bayes estimators, where priors were assumed to be informative.

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Solving an Integral Equation Arising from the Ruin Probability of Long-term Bonus-Malus Systems

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ABSTRACT This article studies in detail the solution of an integral equation due to Rongming et al. [13]. The methods involve complex analysis. As an application, we find the ruin probability of a given Bonus-Malus system in a steady state. We obtain closed form solutions for the ruin probability in certain cases, and we characterize these cases. We give conditions for the Laplace transform of a ruin probability to extend to a meromorphic function in the complex plane, we prove a very general and almost sharp inequality of Lundberg type, and we extend our results to a doubly stochastic situation.

Keywords Bonus-Malus systems; Complex variables; Doubly stochastic systems; Fourier-Laplace transforms; Ruin probability.

1. Introduction and Motivation

The integral equation to be solved can be written as

\[-(\lambda_1 + \lambda_2)\tilde{\psi}(u) + \lambda_1 E(\tilde{\psi}(u+C)) + \lambda_2 \int_0^u \tilde{\psi}(u-x)f_x(x)dx = 0,\]  

(1.1)

where \(\tilde{\psi}(u+C)\) is an expectation value and \(C\) is a given discrete (or continuous) probability distribution. The above equation is further discussed in Rongming et al. [13]. We give some background and motivation before proceeding.

In insurance, a bonus-malus system (BMS) is a system that adjusts the premium paid by a customer according to his individual claim history. A bonus usually is a discount in the premium which is given on the renewal of the policy if no claim is made in the previous year. A
The McDonald Log-logistic Distribution with Applications to Lifetime and Pricing Data

Shujiao Huang                               Broderick O. Oluyede
University of Houston                  Georgia Southern University

ABSTRACT A new multi-parameter class of distributions called McDonald Log-logistic (McLLoG) distribution is proposed. This class of distributions contains several distributions such as beta Log-logistic, exponentiated Log-logistic and Log-logistic as special cases. The hazard, reverse hazard function, mean residual life function and moments are obtained. Lorenz and Bonferroni curves, distribution of order statistics and Rényi entropy are derived. Maximum likelihood estimates as well as asymptotic confidence intervals for the model parameters and simulation study are given. Examples and applications to lifetime and new car pricing data are presented.

Keywords Log-logistic distribution; Maximum likelihood estimation; McDonald Log-logistic distribution.

1. Introduction

The log-logistic distribution is a very useful distribution with applications in several areas including survival analysis, hydrology and economics. There are several generalizations of this distribution including the beta log-logistic, presented by Lamonte [15] following the generator approach introduced by Eugene et al. [9]. Jones [12] developed and presented family of distributions arising from the distribution of order statistics. In this note, we present the McDonald log-logistic distribution and its statistical properties with applications to new car pricing and lifetime data. Some McDonald generalized distributions in the literature include work by Cordeiro et al. [4] on the McDonald extended distributions generalizing the exponential, generalized exponential, Kumaraswamy exponential and beta exponential (Barreto-Souza...
Discriminating between Generalized Exponential and Gamma Distributions

Orawan Supapueng    Kamon Budsaba    Andrei I. Volodin    Pranee Nilkorn
Thammasat University    University of Regina    Silpakorn University

ABSTRACT Generalized Exponential and Gamma distributions are the most popular in analyzing skewed lifetime data. They have many similar properties. Nevertheless they have some different properties, especially when the lifetime data analysis emphasizes the tail of the probabilities. We can observe that it will be more efficient if we can select the correct distribution for a given data. Therefore in this article, we investigate the asymptotic method for distinguishing these two distributions. It is observed that the asymptotic distribution is independent of a nuisance parameter. We perform some numerical experiments to observe that the asymptotic method works for different sample sizes.

Keywords Exponential distribution; Generalized gamma distribution; Generalized invariant property; Lifetime distribution; Separate hypothesis; Statistical testing.

1. Introduction

Nowadays we can observe a dramatic increase of production in different areas of activities. Regardless of what is produced, it is crucial to pay more attention to the reliability of products. Reliability of a device or a product, it is an important indication of its quality. The significant matter about reliability theory is the concept of lifetime distributions. There are many different lifetime distributions because every product will provide different information about its lifetime so that we should be careful and critical in selecting a lifetime distribution to describe lifetime data from a representative sample of units.
Asymptotic Properties and Parameter Estimation Based on Two-Sided Crack Distribution

Wikanda Phaphan Andrei I. Volodin Kamon Budsaba
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ABSTRACT In this paper we propose a new family of two-sided crack distribution. The theoretical properties of the two-sided crack distribution is established. Also, we develop and investigate the method of moments of parameters estimation. A Monte Carlo simulation and real data study are conducted to appraise the performance of the proposed estimators for given sample sizes by using R program for evaluation.

Keywords Birnbaum-Saunders distribution; Inverse Gaussian distribution; Length-biased inverse Gaussian distribution; Maximum likelihood estimators.

1. Introduction

The crack distribution is a positively skewed model, which is widely applicable to model failure times of fatiguing materials. It is also known as the inverse Gaussian mixture distribution, was studied by Jorgensen et al. [11] and Bowonrattanaset and Budsaba [5]. Gupta and Akman [9] proposed the mixture of inverse Gaussian (IG) distribution and length biased inverse Gaussian (LBIG) distribution which given in Jorgensen et al. [11] in a reliability view point, and here called JSW distribution. Gupta and Akman [10] studied the mixture of IG distribution and LBIG distribution in the view of Bayes estimation. Balakrishnan et al. [2] discussed several aspects of the inverse Gaussian mixture distribution which is useful for modelling positive data. Specifically, they discussed transformations, the derivation of moments, fitting of models, and a shape analysis of the transformations. Bowonrattanaset and Budsaba [5] introduced the inverse Gaussian mixture distribution based on re-parametrization model presented in Ahmed et al. [1], and proposed the name crack for this distribution, it will be denoted by \( CR(\lambda, \theta, p) \). They also established some deeper results especially function with rigorous proves. Gupta and Kundu [8] proposed to use the EM algorithm to estimate the unknown parameters of the inverse Gaussian
A New Class of Generalized Log-logistic Weibull Distribution: Theory, Properties and Applications

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ABSTRACT A new generalized distribution called the exponentiated log-logistic Weibull (ELLoGW) distribution is developed and presented. This class of distributions contains exponentiated log-logistic Rayleigh, exponentiated log-logistic exponential, log-logistic Weibull, log-logistic Rayleigh, log-logistic exponential, log-logistic, Weibull, Rayleigh, exponential and several other distributions as special cases. The structural properties of the distribution including hazard function, reverse hazard function, quantile function, moments, conditional moments, mean deviations, Bonferroni and Lorenz curves, entropy and order statistics are derived. The method of maximum likelihood is used to estimate the model parameters and finally, real data examples are discussed to illustrate the applicability of the distribution.

Keywords Exponentiated log-logistic distribution; Generalized distribution; Weibull distribution.

1. Introduction

Lifetime distributions are of tremendous practical importance in probability and statistics as well as other related areas. Weibull and the log-logistic distributions are particularly useful lifetime models and have been widely used for modeling data in a wide variety of areas including reliability, engineering, stochastic processes, survival analysis and renewal theory.

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On Some Compounded Poisson Distributions with Applications to the Pattern of Number of Child Deaths

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Banaras Hindu University                    Rajshahi University

ABSTRACT The intensity and tendency of mortality indicate the standard of development of a society, thus the study of mortality especially child death within the age limit 0-4 years is one of the important area of population sciences. It indicates the lower level of success of health and intervention programs. In this paper an attempt has been made to study the pattern of child deaths experienced by the females. The parameters involved in the model under consideration have been estimated with method of maximum likelihood. A real data set (Bhuyan & Deogratias [2]) has been used to confer the applicability and validity of the proposed models.

Keywords Child death; Maximum likelihood estimates; Probability distribution.

1. Introduction

High rate of death within the age limit 0-1 year particularly in the first month of the birth represents the gross reproductive loss of physical, economical and psychological resources of the females. If one can identify the responsible biological and socio-demographic factor for child death, the health and intervention program can be reformulated and implemented to reduce the intensity of child death. The Infant and child mortality is known as a good and sensitive indicator of development of a nation and impact of government intervention programs and policies. Effective control in reduction of mortality has been one of the remarkable achievements across the world. Child death has been major concern of the researchers and
Some Theoretical Properties and Parameter Estimation for the Two-Sided Length Biased Inverse Gaussian Distribution

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ABSTRACT The new lifetime distribution based on non-classical parametrization model called the two-sided length biased inverse Gaussian distribution is introduced. The physical phenomena of this situation can be explained in the case when a crack develops from two sides. Some statistical properties of the distribution such as reciprocal properties and the first four moments are investigated. The conventional point estimation, method of moment, is developed for estimating the parameters of the distribution together with asymptotic property of the proposed estimators. In order to evaluate the performance of the suggested estimators, Monte Carlo simulation studies are conducted. Additionally, real data sets in a practical setting are used to illustrate the presented estimation method. Concluding remarks and discussions are also provided.

Keywords Asymptotic property; Lifetime distribution; Method of moment estimate; Parametrization; Reciprocal property.

1. Introduction

Lifetime distributions are frequently studied in reliability aspects. It is easy to consider a lifetime or failure time of physical objects such as coins, electric light bulbs, some pieces of machines, etc. They provide useful information on certain practical problems. Since some machines or systems are very important and extremely expensive, this information motivates practitioners to prevent financial or industrial damages occurring after the failure time terminates. One of the interesting views of lifetime distributions in reliability analysis is in the situation when a failure of the object under consideration occurs from a fatigue crack development. The common distributions used in practical applications of this area are Birnbaum-Saunders, inverse Gaussian, and length biased inverse Gaussian [7, 13].
The Double Prior Selection for the Parameter of Rayleigh Lifetime Model under Type-II Censoring

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ABSTRACT This paper covers a comparison of double priors assumed for the parameter of Rayleigh life time model. Instead of single prior for the parameter of the given distribution, sometimes we may have different prior information for the single parameter. Hence it may be beneficial to include all such different information in the estimation of the parameters. Here we have considered three pair of different types of prior distributions for the parameter of the Rayleigh life time model. The results based on double prior distributions are compared with the results based on single prior distribution. Based on a type-II censored data Bayes estimation is carried out under squared error loss function for the three different sets of double prior distribution for future failure time and for the remaining failure times after the first \( r \) failure times observed have been derived. A real life example is taken to demonstrate the results derived.

Keywords Bayes estimation; Bayes risk; Hartigan prior; Inverted gamma distribution; Jeffrey’s prior; Squared error loss function.

1. Introduction

Rayleigh distribution was invented by Lord Rayleigh [1] during a study of acoustical problems. It is widely useful in communication engineering (Dyer and Whisenand [2]), electro vacuum devices (Polovko [3]), and some clinical studies dealing with cancer patients (Battacharya and Tyagi [4]). This distribution was successfully employed as a radio wave power distribution (Siddiqui [5]). It is extremely useful in life testing, reliability performances and survival analysis. In life testing experiments at the end of the experiment the units are lost or destroyed, in such experiments different types of censoring schemes are used. There are two
ABSTRACT  In this article, we develop the maximum likelihood estimation for the three-parameter of the Crack lifetime distribution and also consider the bias-reduction of the estimators obtained from the classical estimation. Moreover, we consider the Bayesian estimation which we provide by assuming an informative priors. The Bayes estimators are obtained from the Gibbs sampling procedure to generate samples from the posterior distribution and also from the Lindley’s approximation method. A simulation study carried out to estimate and compare the various point estimation methods considered.

Keywords  Bayesian estimation; Bias reduction; Bootstrap resampling, Composition method; Crack lifetime distribution; Gibbs sampling; Inverse transform method; Jackknife; Lindley’s approximation; Markov chain Monte Carlo; Maximum likelihood estimation.

1. Introduction

The engineering interpretation of Crack random variable as the time after a crack started to develop in a machine element because of a cyclic or non-cyclic loading until the crack achieves the critical value. At the beginning, it may be a small crack in the machine, but the element could still work. When it achieves the critical point, tolerance exceeds and the element does not work anymore. The three-parameter Crack lifetime distribution had been introduced by Volodin and Dzhungurova [12] as a distribution that is performed by adding weighted parameter and combining the Inverse Gaussian distribution and Length Biased Inverse Gaussian distribution. Thus, the Crack lifetime distribution contains as special cases three known distributions, i.e., the Birnbaum-Saunders distribution, the Inverse Gaussian distribution and the Length Biased Inverse Gaussian distribution. Bowonrattanaset and Budsaba [1] established properties of this distribution. The probability density function of the Crack lifetime distribution is given by:
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