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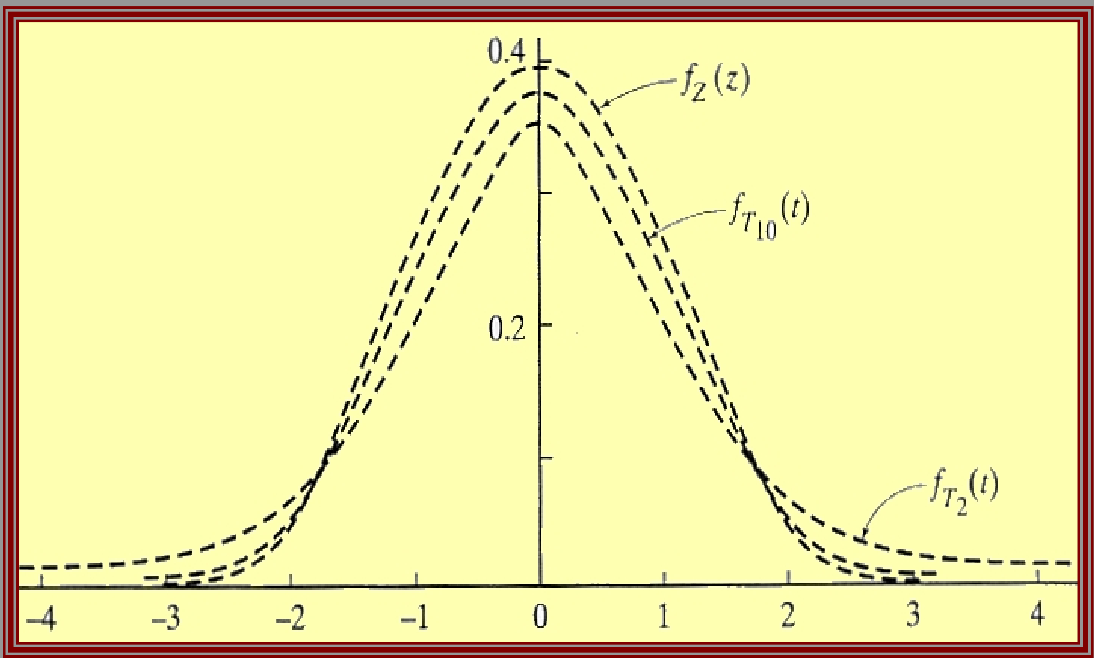
JPSS

A comprehensive journal of probability and statistics
for theorists, methodologists, practitioners, teachers, and others



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Aims and Scope The ***Journal of Probability and Statistical Science*** (***JPSS***, ISSN 1726-3328) is a modified version of the ***Journal of Propagations in Probability and Statistics*** (***JPPS***, ISSN 1607-7083). ***JPSS***, like its predecessor ***JPPS***, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, and any other users of probability and/or statistics. The scope of ***JPSS*** is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of ***JPSS*** to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability and/or statistics.

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Journal of Probability and Statistical Science
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Weak Laws of Large Numbers for Fields of Random Variables in Banach Spaces

Ta Cong Son Dang Hung Thang Phan Viet Thu
VNU University of science

ABSTRACT For an adapted field of random variables $\{X_n, \mathcal{F}_n; n \succeq 1\}$ in a real separable p-uniformly smooth Banach space, some weak laws of large numbers are established. Conditions are provided under which

$$\frac{1}{a_n} \sup_{1 \preceq k \preceq u_n} \sum_{1 \preceq i \preceq k} a_{ni} \left(X_i - E(Y_{ni} \mid \mathcal{F}^*_{i-\alpha(i)}) \right) \to 0 \text{ in probability}$$

and

$$\frac{1}{a_n} \sup_{1 \preceq k \preceq \tau_n} \sum_{1 \preceq i \preceq k} a_{ni} \left(X_i - E(Y_{ni} \mid \mathcal{F}^*_{i-\alpha(i)}) \right) \to 0 \text{ in probability}$$

where $\{\tau_n \leq \infty;\ n \succeq 1\}$ is the field of positive integer-valued random variables and

$$Y_{ni} = X_i I(\| X_i \| \leq b_{ni}).$$

Our results extend the results of Sung *et al*. [10, 11], Quang *et al*. [6, 7] and some other ones.

Keywords α-strong adapted random fields; α-strong* adapted random fields; p-uniformly smooth Banach spaces; Weak laws of large numbers theorem.

1. Introduction

The weak laws of large numbers for arrays of random variables (real-valued or Banach space-valued) are studied by many authors (see, e.g., Adler *et al*. [1], Hong *et al*. [2], Quang *et al*. [6, 7, 8], Wei *et al*. [12]). Recently, Sung *et al*. [10, 11] obtained the weak law of large numbers with random indices for array of random elements, Quang and Huan [6] established

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Marshall-Olkin Exponentiated Generalized Fréchet Distribution and Its Applications

K. K. Jose and Remya Sivadas
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ABSTRACT A generalization of the Marshall-Olkin family of distributions is developed by using geometric compounding. It is illustrated by considering the Marshall-Olkin Exponentiated Generalized Fréchet distribution and various properties including moments, generating function, order statistics are discussed. Estimation of parameters are also considered and the new model is applied to a real data set and the results are verified. Stress-strength reliability analysis and simulation studies are also conducted.

Keywords Exponentiated distributions; Exponentiated Generalized Fréchet distribution, Survival function; Hazard rate function; Marshall-Olkin family of distribution; Stress-strength reliability.

1. Introduction

Gupta *et al*. [7] introduced exponentiated exponential distribution as a generalization of standard exponential distribution. Nadarajah and Kotz [19] proposed the exponentiated gamma, exponentiated Fréchet and exponentiated Gumbel distributions. Cordeiro *et al*. [5] proposed a new class of distributions that extend the exponentiated generalized type distributions. Given a continuous cdf $G(x)$, the cdf of the Exponentiated Generalized (EG) class of distribution is obtained as one having general structure given by

$$F(x) = \{1-[1-G(x)]^{\alpha}\}^{\beta}, \tag{1}$$

where $\alpha > 0$ and $\beta > 0$ are two additional shape parameters. The base line distribution $G(x)$ is a special case of (1) when $\alpha = \beta = 1$. When $\alpha = 1$, we get the exponentiated type distributions. The probability density function of the exponentiated generalized class of distributions is given by

$$f(x) = \alpha\beta[1-G(x)]^{\alpha-1}\{1-[1-G(x)]^{\alpha}\}^{\beta-1} g(x). \tag{2}$$

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A Skew Pareto Distribution on the Real Line

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ABSTRACT We introduce a new probability distribution on the real line, whose restrictions to positive and negative semi-axes are Lomax laws. We present basic properties of this model, which include moments and related parameters, stochastic representations, and divisibility properties. We also briefly discuss inferential issues connected with this model.

Keywords Asymmetric Laplace distribution; Double exponential distribution; Epsilon-skew distribution; Infinite divisibility; Skew double-exponential model; Skewness; Skew-symmetric distribution; Stable distribution; Stochastic representation.

1. Introduction

The normal distribution is perhaps the most widely used probability model in the sciences. However, being a light-tail distribution, this model does not adequately describe power law (fat tailed) or asymmetric empirical distributions arising in mathematical finance and other applications. Although the more general class of stable distributions (see, e.g., [49]) does not suffer from these drawbacks, it is not a convenient family to work with for a practitioner, since its basic characteristics such as the probability density function (PDF) and cumulative distribution function (CDF) are not available in closed forms. Moreover, the power-law tail parameter of a stable distribution is restricted to the range (0, 2). With these limitations of stable laws in mind, we introduce a new class of skew distributions on the real line, by extending the Lomax distribution (see [42]) with the PDF

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Weighted Generalizations of the Generalized Rayleigh and Related Distributions with Applications

Broderick O. Oluyede   Oluseyi Odubote   Xueheng Shi
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ABSTRACT In this paper, weighted generalizations of the generalized Rayleigh distribution is proposed and studied in detail. We present the statistical properties of the weighted generalized Rayleigh distribution with the weight function

$$w(x) = x^k \exp\{tx^\beta\} F^i(x) \overline{F}^j(x)$$

when $i = j = 0$. In particular, we derive the probability density function (pdf), cumulative distribution function (cdf), hazard function, reverse hazard function, moments, coeff-cients of variation, skewness and kurtosis. Shannon entropy, Renyi entropy, β-entropy, generalized entropy and Fisher information are derived. Maximum likelihood estimates of the model parameters are also presented. Test procedures for weightedness including length-biasedness concerning the Rayleigh, generalized Rayleigh and weighted generalized Rayleigh models are developed. Real data examples are given to illustrate the usefulness of the proposed distribution.

Keywords Generalized distribution; Weighted generalized Rayleigh distribution; Moments; Uncertainty measures.

1. Introduction

Weighted distributions occur naturally in research related to reliability, biomedicine, ecology and several other areas and are of tremendous practical importance in probability and statistics. These distributions arise as a result of observations generated from a stochastic process and recorded with some weight function. Rao [13] identified the various sampling situations that can be modeled by what he called weighted distributions. Patil and Rao [11, 12]

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Generalized Compound Gamma Model with an Application to Sea Clutter Data

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ABSTRACT In recent time, various efforts have been made to develop a new class of generalized model that extends well-known families of distributions and at the same time it provides great flexibility in modeling data. Generalizing or mixing is a standard technique for the interpretation or construction of distribution, which gives a rich class of models for applications in various fields. In this article, a new family of distribution, which is a natural composition of generalized gamma and two parameter gamma model is presented as an alternative to several compound gamma models. The new model contains several important compound gamma models, such as Weibull-gamma, gamma-gamma and K-distribution etc. Statistical properties such as moments and moment generating function have been worked out for generalized compound gamma model. The connection between the generalized compound gamma model and the various distributions such as generalized logistic model and type-2 beta distribution are presented. Application of the proposed model in the wireless fading channel and sea clutter data modeling are also discussed.

Keywords Modified Bessel function of third kind; Mellin transform; Compound models; Generalized logistic model; Compound gamma model.

1. Introduction

In recent years, compound gamma models have a surge of consideration in statistical literature. This type of statistical models have been introduced and studied in the last few years by compounding continuous or discrete distributions. For a review of the literature we can refer to Johnson *et al*. ([13], [14]). Compounding is considered as a flexible and powerful statistical modeling technique. Additional advantage of compound models is that these models cover more practical situations in engineering fields, see Nadarajah and Kotz [20]. The method of compounding or mixing is a standard tool for the interpretation or formation of distribution, for

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On Approximating Ruin Probability of Double Stochastic Compound Poisson Processes

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University of New Brunswick

ABSTRACT Consider a surplus process which both of collected premium and payed claim size are two independent compound Poisson processes. This article derives two approximated formulas for the ruin probability of such surplus process, say double stochastic compound poisson process. More precisely, it provides two mixture exponential approximations for ruin probability of such double stochastic compound poisson process. Applications to long_term Bonus_Malus systems and a heavy-tiled claim size distribution have been given. Improvement of our findings compared to the Cramér-Lundberg upper bound has been given.

Keywords Ruin probability; Double stochastic compound Poisson processes; Bonus-Malus system; Heavy-tailed distributions; Laplace transforms; Cramér-Lundberg upper bound.

1. Introduction

Consider double stochastic compound Poisson process

$$U_t = u + \sum_{i=1}^{N_1(t)} C_i - \sum_{j=1}^{N_2(t)} X_j , \qquad (1)$$

where $C_1, C_2, \cdots$ and $X_1, X_2, \cdots$ respectively, are two independent i.i.d. random samples from independent random premium C and random claim size X; two independent Poisson processes $N_1(t)$ and $N_2(t)$ (with intensity rates λ_1 and λ_2) are, respectively, standing for claims and purchase request processes, and u represents initial wealth/reserve u of the process. Moreover, suppose that non-negative and continuous random premium C and claim X; respectively, have

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Second-Order Test Allocation for Estimating Reliability of Series Systems with Two Components

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ABSTRACT In this paper, we adopted a Bayesian approach to estimate the reliability of a series system with two components under the squared error loss. We derived an asymptotic second-order lower bound for the Bayes risk of a sequential procedure that allocates M test cases to one component and t-M to the other component, where M is determined according to a sequential design and t denotes the total number of test cases. A fully sequential sampling scheme is proposed and Monte Carlo simulation has been implemented.

Keywords Second-order test allocation; Reliability; Series system; Bayes risk; Monte Carlo simulation.

1. Introduction

Overestimating software reliability could have disastrous consequences, especially in critical systems where the tolerance for failure can be on the order of 10^{-3} or smaller [4]. On the other hand, estimates that closely approximate reality can reduce risk and decrease the cost of software development. Some studies ([8], [9]) have been done in the past to estimate parallel systems, where software is partitioned into several independent subdomains ([1], [10], [11]). In this paper, however, we will estimate the reliability of a series system with two components.

Let $\mathcal{P}_1$, $\mathcal{P}_2$ be two components in a series system with associated values θ and ω, respectively, where θ and ω is the conditional reliability of a use case or a test case [5], on condition that it was randomly chosen within two components. Within each component, each test case has an equal chance to be selected. Note that θ and ω are unknown parameters [3]. Then the reliability of the series system with two components is defined as

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