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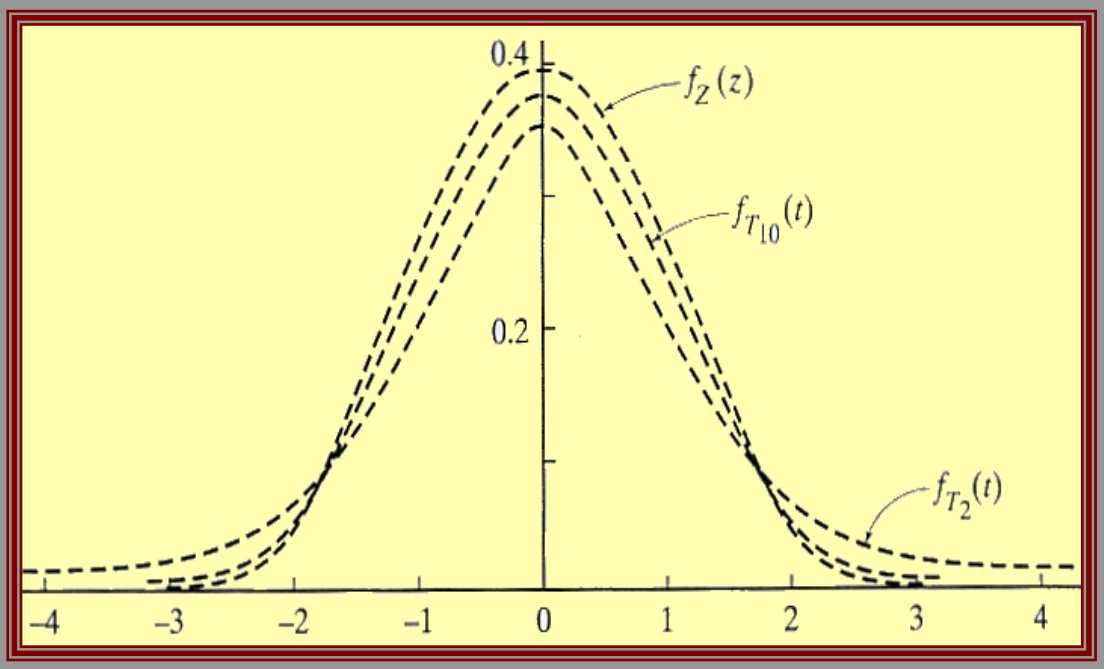
A comprehensive journal of probability and statistics
for theorists, methodologists, practitioners, teachers, and others



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Journal of Probability and Statistical Science
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An Approximation to Duration Distribution of the Minimum of Two Gaussian Processes

M. T. Alodat
Qatar University

ABSTRACT We derive an approximation to the duration distribution of the excursion for the point wise minimum of two smooth, independent and stationary Gaussian random processes above a high threshold. The mean and the variance are calculated for this distribution. Also we conduct a simulation study to compare the approximation with the exact distribution based on simulations from a Gaussian process example. Moreover, we apply our findings to real data from power engineering.

Keywords Gaussian process; Point wise minimum; Duration of an excursion.

1. Introduction

Gaussian processes are widely used to model many random responses in engineering. For example, in communication engineering, Gaussian processes are flexible models for the load of a communication system. An extreme value of a Gaussian load indicates that the communication system is unavailable. So the probability that the load of a system exceeds a given threshold is used as a measure for the communication quality. Another measure for systems quality is the time of unavailability, i.e., the period of time that the load spends above a given threshold u, say, or the duration that the load spends above the threshold u after an upcrossing of u (see Figure 1 for upcrossings, downcrossings and and durations). Other example arises in oceanography, where the load of a ship, which is undertaking a voyage over a period of time, is modelled as a Gaussian process. So if $X(t)$ is the load of a ship, then the duration of $X(t)$ is interpreted as the duration of a risk on the ship (Aalbers *et al*. [1]). Also, in electrical engineering as well as in digital communication systems, the fade duration is the length of interval of time that a signal spends below or above a given threshold. Hence, the average fade duration is useful to determine the severity fading over time (Rappaport [13]). It is well-known that finding the exact

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M. T. Alodat is affiliated to the Department of Mathematics, Statistics and Physics at Qatar University, Doha, Qatar; email: alodatmts@yahoo.com.

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Asymptotic Normality of the Recursive Kernel Estimate of Conditional Cumulative Distribution Function

Bouadjemi Abdelkader
Université de Sidi Bel Abbès

ABSTRACT In this paper, we introduce a new nonparametric estimator of the conditional cumulative distribution of a scalar response variable Y given a functional random variable X. Our estimate is based on recursive approach. We prove under general conditions, the asymptotic normality of the constructed estimate.

Keywords Recursive estimation, Conditional cumulative distribution, Functional random variables, Semi-metric space, small balls probability.

1. Introduction

Nonparametric estimation in functional data is a challenging thematic in functional data analysis that has generated much interesting researchs (see, Ferraty and Vieu [7] or Ferraty and Romain [4] for a deeper discussion on this area). In this topic the nonparametric estimation of the conditional distribution function has great importance. Indeed, this nonparametric model has the advantages of characterizing the whole conditional law of the considered random variables, which allow to obtain the conditional density functions, the conditional hazard function and the conditional quantile functions. This paper deal with the nonparametric estimation of the cumulative distribution function of scalar response given functional random variables by using the recursive estimation method.

In recent years, there is a vast literature devoted to the study of conditional models in nonparametric functional statistics. The first results, in this topic, dates back to 2006. It was given by Ferraty *et al*. [6]. In this pioneer work the authors gave the almost complete convergence of the kernel estimator of the conditional distribution function, the conditional density and its derivatives. The asymptotic normality of the cumulative distribution function

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Bouadjemi Abdelkader is affiliated to Laboratoire de Statistique et Processus Stochastique, Université de Sidi Bel Abbès, BP 89 Sidi Bel Abbès 22000, Algeria; email: bouadjemi@gmail.com.
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Autoregressive Processes with Multivariate Laplace Marginals

K. K. Jose and Manu Mariam Thomas
St. Thomas College Palai, Mahatma Gandhi University

ABSTRACT The multivariate Laplace distributions are considered and their properties are discussed. Autoregressive processes with multivariate symmetric and asymmetric Laplace marginal distributions are developed. Multivariate Geometric Laplace distributions in the symmetric and asymmetric cases are introduced and the corresponding autoregressive processes are discussed. Stability properties of multivariate Laplace distributions are discussed. Some of the applications are also studied.

Keywords Autoregressive processes; Geometric infinite divisibility; Multivariate asymmetric Laplace distribution; Multivariate geometric Laplace distribution; Self-decomposability; Time series modelling; Vector auto-regressive process.

1. Introduction

The developments in the area of multivariate skew distributions started with Azzalini and Dalla Valle [2] and Azzalini and Capitanio [1]. Ernst [4] introduced multivariate extension of symmetric Laplace distributions via an elliptic contouring. Mathai [17] also consider some generalizations of Laplace distributions and discussed quadratic forms in multivariate Laplace random vectors. The classical univariate Laplace distribution, also known as the double exponential, is introduced by Laplace in 1774. The most outstanding characteristics of this distribution are that it is unimodal and symmetric. The Laplace distribution has found applications in a variety of engineering areas that range from image and speech recognition to ocean engineering. This distribution is often used for modeling phenomena with "heavier than normal tails".

Asymmetric Laplace(AL) distribution is the most appropriate generalization of the classical Laplace law, which arises naturally as limiting distributions in random summation scheme.

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Authors of this article are affiliated to the Department of Statistics in St. Thomas College at Mahatma Gandhi University, Arunapuram P.O, Palai, Kerala 686 574, India; email address of K. K. Jose (corresponding author): kkjstc@rediffmail.com.

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Distributional Properties of Curtate Future Life Time

HarmanPreet Singh Kapoor and Kanchan Jain
Panjab University

ABSTRACT For an individual aged x, some characteristics of the distributions of curtate future lifetime have been studied. Assuming different distributional forms for a new born's lifetime, the expressions for moments, coefficients of skewness and kurtosis of curtate future lifetime distributions have been derived. The maximum likelihood estimation (MLE) of the parameters has been explored. Simulations have been carried out to find the ML estimates and the corresponding root mean square errors.

Keywords Actuarial; Future lifetime; Curtate future lifetime; Survival probability; Curtate distribution.

1. Introduction

In insurance sector, the curtate future lifetime plays a significant role in the assurance contracts and discrete life annuities when the benefit is payable at the end of year of death of the claimant/insured. Many authors have studied the properties of complete future life time, termed as residual/remaining life in reliability theory and survival analysis [10]. For determining the premium of an insured, the insurer's interest is not only in the complete future lifetime but also in the individual's curtate future lifetime. However, the statistical study of curtate future lifetime has not attracted the attention of researchers so far and that has motivated us to investigate some characteristics of distributions of curtate future lifetime.

Let the random variable X be the age at death for a new-born child. For any positive x,

$$\overline{P}_x = P(X > x)$$

denotes the survival probability that a new born attains at least age x and

$$P_x = P(X \le x).$$

The notation (x) denotes a life aged x whose complete future lifetime is written as $T(x)$.

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Authors of this article are affiliated to the Department of Statistics at Panjab University, Chandigarh-160014, India; email address of Kanchan Jain (corresponding author): jaink14@gmail.com.

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Construction of Higher Associate Class Partially Balanced Incomplete Block Designs Using Factorial Combinations

Davinder Kumar Garg
Punjabi University

Gurinder Pal Singh
Bhai Gurdas Institute of Engineering and Technology

ABSTRACT A new series of s ($s \geq 2$) associate class Partially Balanced Incomplete Block (PBIB) designs have been constructed by using factorial combinations. In this class, we consider 's' different factors ($s \geq 2$) simultaneously and each factor has two levels . We consider the correspondence between factors with respect to their levels in all possible factorial combinations for the purpose of constructing new Partially Balanced Incomplete Block Designs. Efficiencies of the series of designs along with an association scheme, illustration of construction and a table for comparing these designs with respect to existing designs are also given in this paper.

Keywords Partially balanced incomplete block designs; Factorial combinations.

1. Introduction

In Balanced Incomplete Block Designs, the problem arises when we have a large number of experimental units used in every block, because practically, it increases our block size. Due to that the homogeneity affected among the experimental units in a block. So, it reduces the efficiency of the given Balanced Incomplete Block (BIB) design. To solve this problem, we have attempted to introduce the partially balanced incomplete block designs which are very useful in such type of situations and remain highly efficient. As we know every PBIB design has its own association scheme.

In the available literature, two associate class association schemes for instance, group divisible, triangular, latin square , two associate cyclic and many more association schemes have been introduced by various authors, e.g., Bose and Nair [1], Bose and Shimamoto [2] and others. As for higher associate class association schemes, Vartak [13] introduced a three class association scheme namely 'rectangular association scheme'. Tharthare [12] introduced right

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Davinder Kumar Garg is an Associate Professor in the Department of Statistics at Punjabi University, Patiala-147002, India; e-mail: dkgarg_stat@yahoo.co.in. Gurinder Pal Singh is a Lecturer at Bhai Gurdas Institute of Engineering and Technology, Sangrur -148001, India; e-mail: gurinderpal13@yahoo.com.
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On the Semi Mittag-Leffler Distribution

Davis Antony Mundassery — *Christ College*
K. Jayakumar — *University of Calicut*

ABSTRACT The aim of the present work is to introduce multivariate forms of the semi Mittag-Leffler distribution. Their characterizations are obtained using the distribution of geometric sums. First order stationary autoregressive models are developed with marginals that have multivariate semi Mittag-Leffler distribution. A generalized form of the multivariate semi Mittag-Leffler distribution is introduced and characterized.

Keywords Autoregressive processes; Geometric sums; Mittag-Leffler distribution; Semi Mittag-Leffler distribution.

1. Introduction

A random variable X with positive support is said to follow a semi Mittag-Leffler distribution with exponent α, $0<\alpha\le 1$ if its Laplace transform is

$$\phi(\lambda)=\frac{1}{1+\eta(\lambda)}$$

where $\eta(\lambda)$ satisfies the functional equation

$$\eta(\lambda)=\frac{1}{p}\eta(p^{1/\alpha}\lambda),$$

$0<p<1$. A solution of this functional equation is

$$\eta(\lambda)=\lambda^{\alpha}h(\lambda)$$

where

$$h(\lambda)=\frac{1}{p}h(p^{1/\alpha}\lambda).$$

(for proof see, Kagan *et al*. [6]). When $h(\lambda)=1$, we get the Mittag-Leffler distribution introduced

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Davis Antony Mundassery is affiliated to the Department of Statistics at Christ College, Irinjalakuda, Kerala 680 125, India; email: dmundassery@yahoo.com. K. Jayakumar is affiliated to the Department of Statistics at University of Calicut, Calicut University P.O., Kerala 673 635, India.

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A Class of Strong Deviation Theorems for the Sequences of Arbitrary Random Variables with Respect to *m*th-order Markov Chains

Kang-kang Wang, Yuli Chen, Zhenghui Li, Jiao Deng, Jing Cai, and Fangjuan Cheng
Jiangsu University of Science and Technology

ABSTRACT In this paper, the notion of the limit relative log-likelihood ratio is introduced. A subset of the sample space is given by restricting the likelihood ratio, and on this subset a class of limit theorems, represented by inequalities, for the sequence of random variables with respect to the *m*th-order Markov chain on the random selection system are obtained. As corollaries, some strong deviation theorems for the multivariant state frequency of the sequence of arbitrary sequence of random variables are obtained.

Keywords Strong deviation theorem; Random selection; Likelihood ratio; Multivariant state frequency.

1. Introduction

Suppose $\{X_n, n \geq 1\}$ is a sequence of random variables defined on any probability space $(\Omega, \mathsf{F}, \mathsf{P})$ taking values in the alphabet set $S = \{ s_1, s_2 \cdots \}$ with the joint distribution

$$P(X_0 = x_0, \ldots X_n = x_n) = g(x_0, \cdots, x_n), \ X_i \in S, \ 0 \leq i \leq n. \tag{1}$$

Let Q be another probability measure on (Ω, F) and $\{X_n, n \geq 1\}$ be an *m*th-order non-homogeneous Markov chain on the measure Q with the *m*-dimensional initial distribution and the *m*th-order transition probabilities as follows:

$$q_o(i_1, \cdots, i_m) = Q(X_0 = i_1, \cdots, X_{m-1} = i_m). \tag{2}$$

$$q_n(j \mid i_1, \cdots, i_m) = Q(X_n = j \mid X_{n-m} = i_1, \cdots, X_{n-1} = i_m). \tag{3}$$

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Authors of this article are affiliated to the School of Mathematics and Physics at Jiangsu University of Science and Technology, Zhenjiang, 212003 China; email address of Kang-kang Wang (corresponding author): wkk.cn@126.com.
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Availability of Systems Subject to Harware and Software Failures

M. S. Moustafa
The American University in Cairo

ABSTRACT This paper presents continuous time Markov chains (CTMC) model to obtain availability and the mean time to first failure (MTTFF) of a parallel system subject to independent hardware and software failures. Failures are removed by a finite number of repairmen. The results are illustrated by examples.

Keywords Hardware and software failures; Finite number of repairmen; Availability; MTTFF.

1. Introduction

The availability of computing systems can be improved by hardware and software redundancy techinques. A survey article discussing the redundancy issues in both hardware and software of computing systems is given in [1]. Most of the research in the area of the availability of computing system has been limited to either the hardware failures alone or the software ones alone. The availability of combined hardware and software is a difficult problem, see [2]. Since 1981 when Goel and Soenjoto [3] published their paper which has considered the performance of combined software and hardware system, many studies such as [4], [5], [6], [7] and [8] have delt with combined availability models including both hardware and software. Laprie [4] has obtained the dependability of a single host system during its operational life by considering a Markov model for hardware and software system assuming that both the hardware and the software have exponential failure and repair processes. Lai *et al.* [8] have extended the work of [4] to two-host and ***N***-host systems assumingthat software failure is independent of hardware failure and there is a single repairman. They gave the Kolomogorov's differential equations of two-host and ***N***-host systems and obtained numerically the transient availability of a triple-host system. No closed form solutions of the steady state probability were

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M. S. Moustafa is affiliated to the Department of Mathematics and Actuarial Science at The American University in Cairo, Cairo, Egypt; e-mail: mmostafa@aucegypt.edu.

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